Magnetic fields driven by tidal mixing in radiative stars

J. Vidal\textsuperscript{1}, David Cébron\textsuperscript{1}, R. Hollerbach\textsuperscript{2}, N. Schaeffer\textsuperscript{1}

\textsuperscript{1}\textit{ISTerre (CNRS, Univ. Grenoble Alpes)}
\textsuperscript{2} Univ. of Leeds

Montpellier, 28 Mars 2018
Stellar context

![Diagram showing mass and time relationship with convective and radiative layers.](image)

\[ M_\odot = \text{Solar mass} \]
Stellar context

Mass $M_*$

Time

ZAMS

Birth

8 $M_\odot$

Convective layer

Radiative layer

1.8 $M_\odot$

$(M_\odot = \text{Solar mass})$

Sun

David Cebron
Stellar context

Mass $M_*$

- 8 $M_\odot$
  - Convective layer

- 1.8 $M_\odot$
  - Radiative layer

Time

Birth

ZAMS

Vega star
($M_* = 2.15 M_\odot$)

Sun

($M_\odot = $ Solar mass)
Magnetism of intermediate mass, radiative stars

10% with surface magnetic field $B_0$

**Strong fields** $B_0$ (e.g. Ap stars)
- Typical strength: $10^2 - 10^4$ G
- Often **static** fields
- Surface fields $B_0$ with a **fossil** origin

**Ultra-weak fields** $B_0$ (e.g. Vega)
- Typical strength: $0.1 - 1$ G
- Often **dynamical** fields
- Surface fields $B_0$ generated by **dynamos**?

Tidal dynamos for Vega-like magnetism?
Tidal instability

Tides $\rightarrow$ Elliptical streamlines

Deformation of all layers
Tidal instability

Tides → Elliptical streamlines

Elliptical or tidal instability!

Deformation of all layers

Magnetic field

Cébron & Hollerbach (2014)
Tidal instability

Tides $\rightarrow$ Elliptical streamlines $\rightarrow$ Turbulent flows

Deformation of all layers

$\Omega$

$\Omega_{\text{def}}$

Elliptical or tidal instability!

Magnetic field

• **OK** for neutral or convective fluids  
  (Cebron et al. 2010)

• But **tidally driven dynamos in stably stratified fluids?**
  $\rightarrow$ radiative stars, lunar core, Encelade, Earth’s liquid core stratified layer, etc.

Cébron & Hollerbach (2014)
Stable stratification

• (Dimensional) Brunt-Väisälä frequency $N_0$

$$N_0^2 = -\alpha \nabla T \cdot g$$

($\alpha$: Coeff. of thermal expansion, $g = -\nabla \Phi$: gravity)

- $N_0^2 > 0 \implies$ **stably stratified**, 
- **Internal** gravity waves.
Stable stratification

- (Dimensional) Brunt-Väisälä frequency $N_0$
  \[ N_0^2 = -\alpha \nabla T \cdot g \]
  ($\alpha$: Coeff. of thermal expansion, $g = -\nabla \Phi$: gravity)
  - $N_0^2 > 0 \implies$ stably stratified,
  - Internal gravity waves.

- Growth rate elsewhere? Dynamo?

- Hypothesis:
  - Boussinesq fluid
  - Barotropic & linear gravity

Barotropic

\[ \Phi (g = -\nabla \Phi) \]

\[ \nabla T \times \nabla \Phi = 0 \]

- No baroclinic instability,
- Growth rate reduced at the poles & equator.
Mathematical formulation

- Basic state: $U_0, T_0, g$.
- Scales: $R_*, \Omega_s^{-1}, \Omega_s^2 R_*/(\alpha g_0)$ and $R_* \Omega_s \sqrt{\mu_0 \rho_*}$.
- Dimensionless numbers
  \[ E_k = \frac{\nu}{\Omega_s R_*^2} \leq 10^{-16}, \quad 10^{-6} \leq Pr = \frac{\nu}{\kappa} \leq 10^{-4}, \quad 10^{-8} \leq Pm = \frac{\nu}{\eta_m} \leq 10^{-4}, \]

Boussinesq equations of the perturbations $(u, \theta, B)$ in the inertial frame

\[
\begin{align*}
\frac{\partial u}{\partial t} &= -(u \cdot \nabla) U_0 - (U_0 \cdot \nabla) u - (u \cdot \nabla) u - \nabla p + E_k \nabla^2 u - \theta g + (\nabla \times B) \times B, \\
\frac{\partial \theta}{\partial t} &= -(U_0 \cdot \nabla) \theta - (u \cdot \nabla) T_0 - (u \cdot \nabla) \theta + \frac{E_k}{Pr} \nabla^2 \theta, \\
\frac{\partial B}{\partial t} &= \nabla \times (U_0 \times B) + \nabla \times (u \times B) + \frac{E_k}{Pm} \nabla^2 B,
\end{align*}
\]

\[ \nabla \cdot u = \nabla \cdot B = 0. \]
Numerical method

3 codes: COMSOL, YALES2, NEK5000

Various codes: MAGIC, ASH, XSHELLS, PARODY, RAYLEIGH

Spectral codes are more efficient & reliable for dynamo computations.

in red: open-source and freely available code
Numerical method

Ellipsoidal geometry

Spherical geometry

Schaeffer (2013, 2017)

- **Open-source** code (https://bitbucket.org/nschaeff/xshells),
- Pseudo-spectral code with finite differences and spherical harmonics,
- Simulations **massively parallel** supercomputers (e.g. OCCIGEN): $2 \times 10^6$ h.c.

Various codes: MAGIC, ASH, XSHELLS, PARODY, RAYLEIGH

World fastest spherical dynamo Boussinesq code
(benchmark of Matsui+16)

3 codes: COMSOL, YALES2, NEK5000

Spectral codes are more efficient & reliable for dynamo computations.
Radiative stars: idealized model \cite{Vidal2018MNRAS}.

- **Fixed** $E_k = 10^{-4}$, $Pr = 1$ and $\Omega_0 = 0$,
- **Control** parameters

\[ \frac{N_0}{\Omega_s} \text{ and max. tidal ellipticity } \epsilon \ll 1, \]

- **Basic** state: $U_0(\Psi_0)$, $T_0(\Psi_0)$, $g(\Psi_0)$,
- **Barotropic** state ($g \times \nabla T_0 = 0$),
- **BC**: Stress-free, fixed temperature and electrically **insulating**.
Hydrodynamic instability \hspace{1cm} (Vidal et al. MNRAS, 2018)

- \( E_k = 10^{-4}, \quad Pr = 1, \quad \Omega_0 = 0, \)
- **Onset** at \( \epsilon_c = 0.054 \quad (N_0/\Omega_s = 0) \)
Hydrodynamic instability

• $E_k = 10^{-4}$, $Pr = 1$, $\Omega_0 = 0$,
• Onset at $\epsilon_c = 0.054$ ($N_0/\Omega_s = 0$)
Hydrodynamic instability (Vidal et al. MNRAS, 2018)

- $E_k = 10^{-4}$, $Pr = 1$, $\Omega_0 = 0$,
- Onset at $\epsilon_c = 0.054$ ($N_0/\Omega_s = 0$)

Total temperature

$\epsilon = 0.2$

$N_0/\Omega_s = 0.5$
Hydrodynamic instability  

(Vidal et al. MNRAS, 2018)

- \( E_k = 10^{-4}, Pr = 1, \Omega_0 = 0 \),
- **Onset** at \( \epsilon_c = 0.054 \) (\( N_0/\Omega_s = 0 \))

\[
\begin{align*}
\epsilon = 0.2 \\
N_0/\Omega_s = 0.5
\end{align*}
\]

\[
\begin{align*}
\epsilon = 0.2 \\
N_0/\Omega_s = 1
\end{align*}
\]
MHD simulations (Vidal et al., MNRAS, 2018)

- Basic flow $U_0$ not dynamo capable (for magnetic Prandtl numbers $Pm \leq 5$),
- Integration over one magnetic diffusive time.
Basic flow $U_0$ not dynamo capable (for magnetic Prandtl numbers $Pm \leq 5$),
Integration over one magnetic diffusive time.

Dynamo for $Rm > 2000$ & $Pm \sim 1$
MHD simulations (Vidal et al., MNRAS, 2018)

- Basic flow $U_0$ not dynamo capable (for magnetic Prandtl numbers $Pm \leq 5$),
- Integration over one magnetic diffusive time.

Dynamo for $Rm > 2000$ & $Pm > 1$

Saturated dynamo
Extrapolation (Vidal et al., MNRAS, 2018)

- Similar to convective scalings,
- Typical surface field strength

\[ B_0 = \frac{3}{2} \sqrt{\frac{3\mu_0}{4\pi}} \frac{R_*^{5/2}}{M_*^{1/2}} \Omega_s \frac{m}{D^3} \left| 1 - \Omega_0 \right|, \quad 10^{-3} \leq \delta \leq 10^{-2} \]
Extrapolation (Vidal et al., MNRAS, 2018)

- Similar to convective scalings,
- Typical surface field **strength**

```
B_0 = \delta \frac{3}{2} \sqrt{\frac{3\mu_0}{4\pi}} \frac{R^5/2_* \Omega_s m}{M^{1/2} D^3} |1 - \Omega_0|
```

**Vega**

- **Measure:** 0.6 ± 0.3 G
- **Theory:** 1 - 1.5 G

*Boehm et al., 2015*
Perspective: tidal mixing of fossil fields

Ongoing collaboration with Dr. Evelyne Alecian (IPAG) & Dr. Asif ud-Doula (Dunmore)

Enhancing of Ohmic decay!
1) Tides can lead to mixing & dynamos in stratified fluids
   - For moderate stratification \[(N/\Omega < 10, \text{ see Vidal+18})\]
   - For large enough tides

2) Tides can accelerate the fossile field (Ohmic) decay via the mixing
Thank you!
Inertial instabilities

Restoring force
(Coriolis, buoyancy, Lorentz force, etc.)

Waves, modes
(inertial, internal, Alfven, etc.)

\[ \mathbf{V} \times \mathbf{V} \]
Inertial instabilities

Restoring force
(Coriolis, buoyancy, Lorentz force, etc.)

Waves, modes
(inertial, internal, Alfven, etc.)

Periodic modulation

Parametric resonance
(tidal or precession instability, Faraday instability, etc.)

\[ \sim \cos[m \, \theta(t)] \]

Precession topographic instability \( \sim \) Tidal instability : similar mechanism
=> inertial instability (parametric resonance)