Magnetic fields driven by tidal mixing in radiative stars

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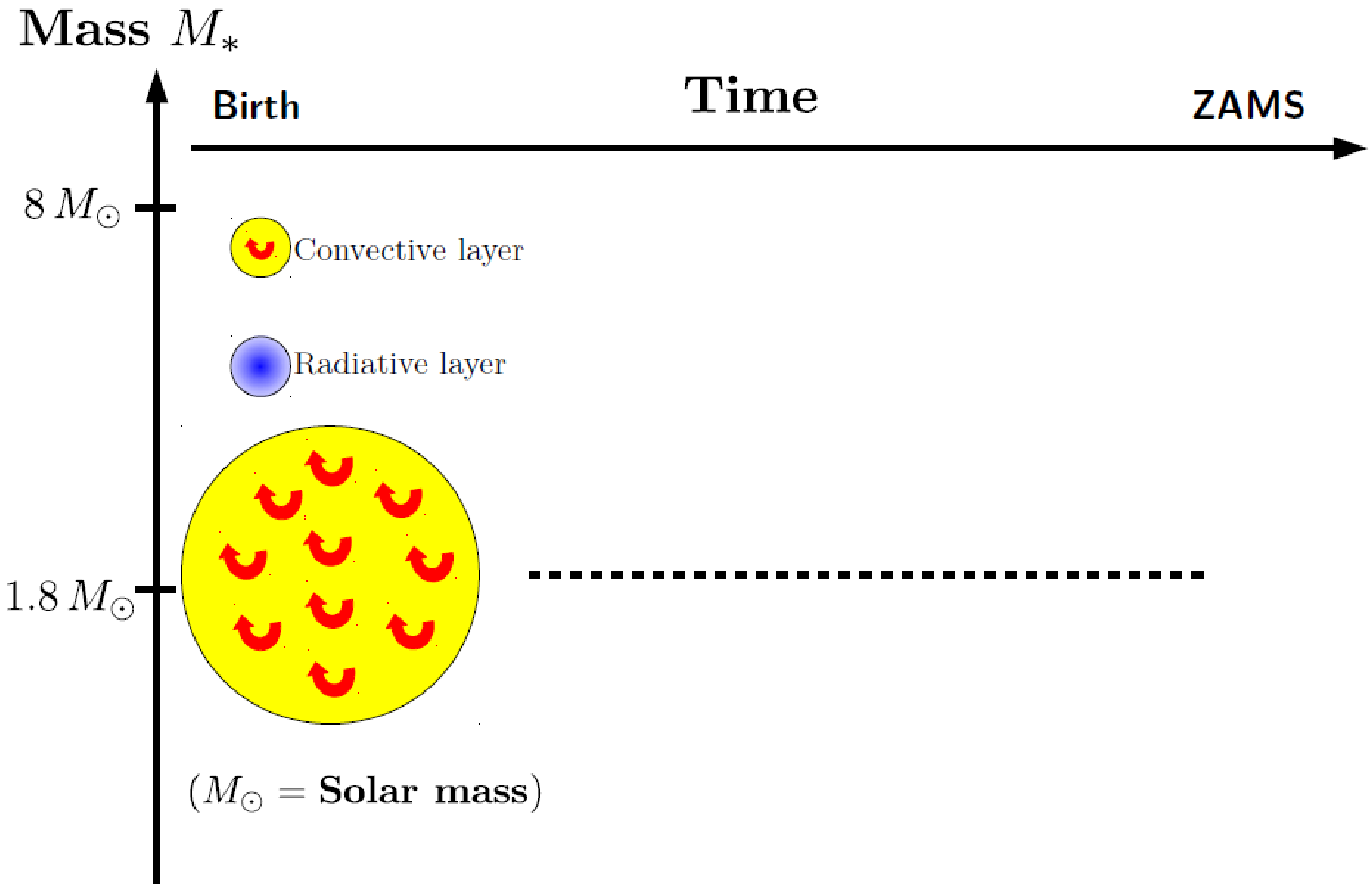
Montpellier, 28 Mars 2018

Stellar context



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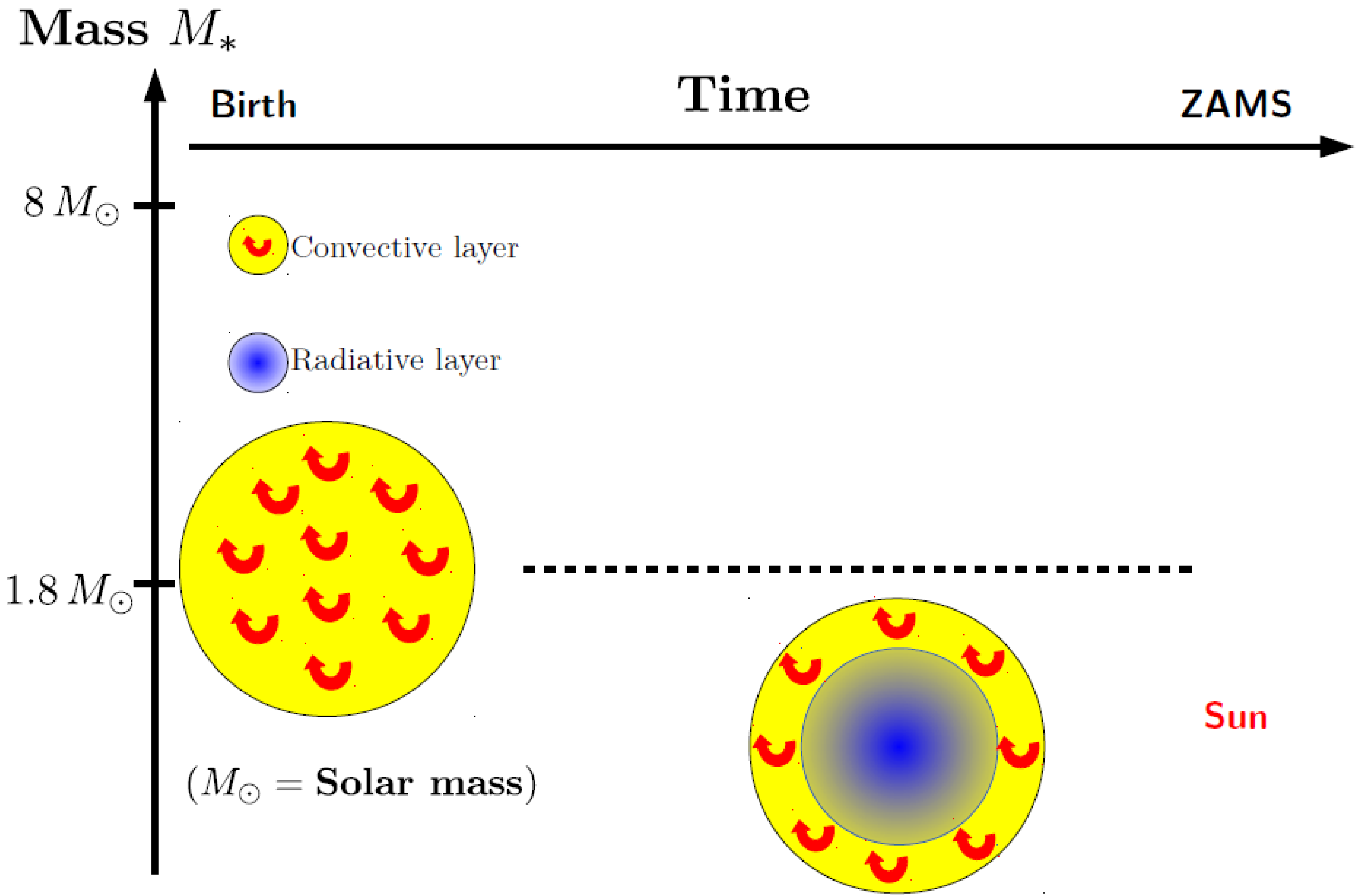

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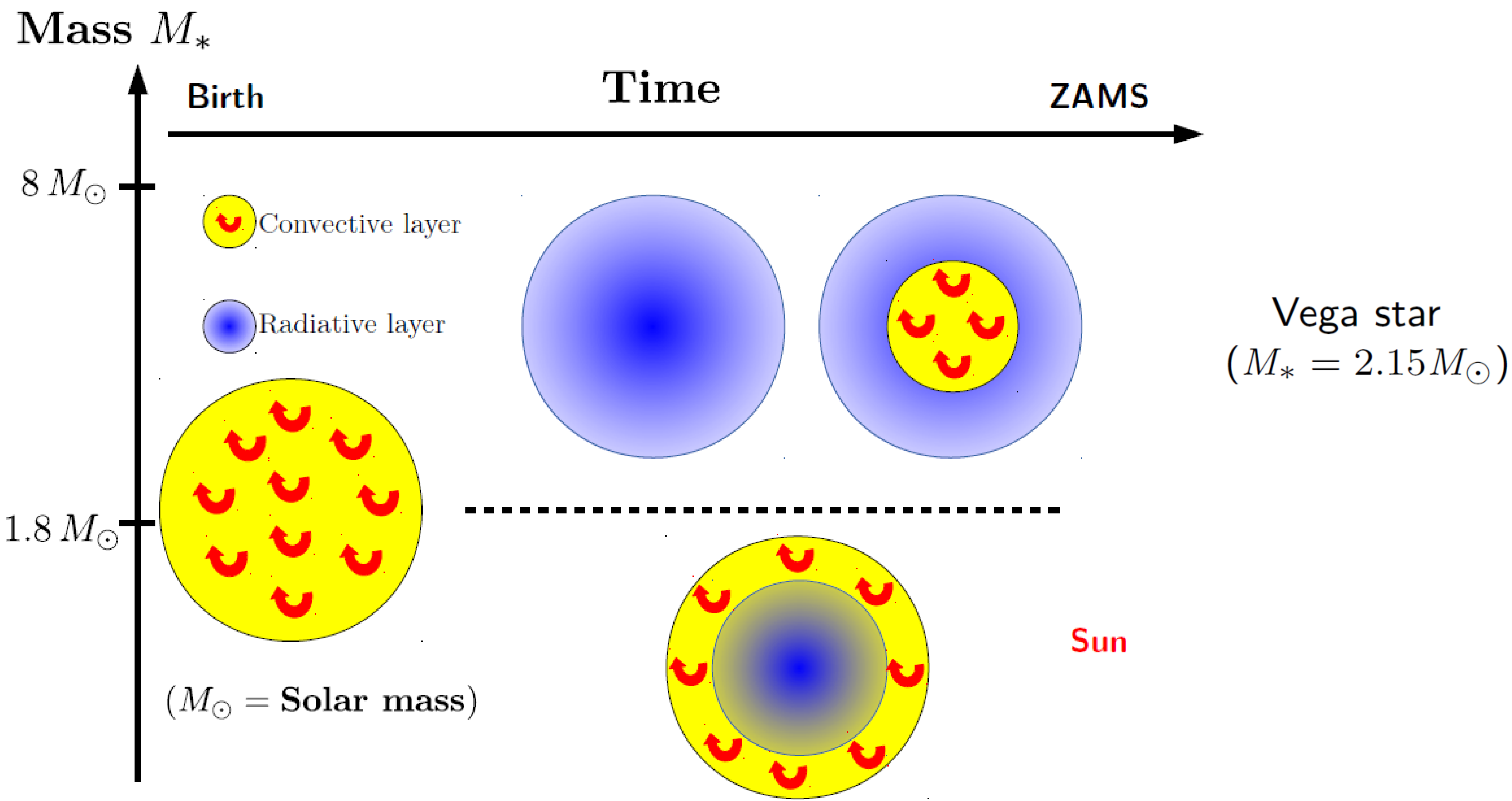


Magnetism of intermediate mass, radiative stars

10% with surface magnetic field B_0

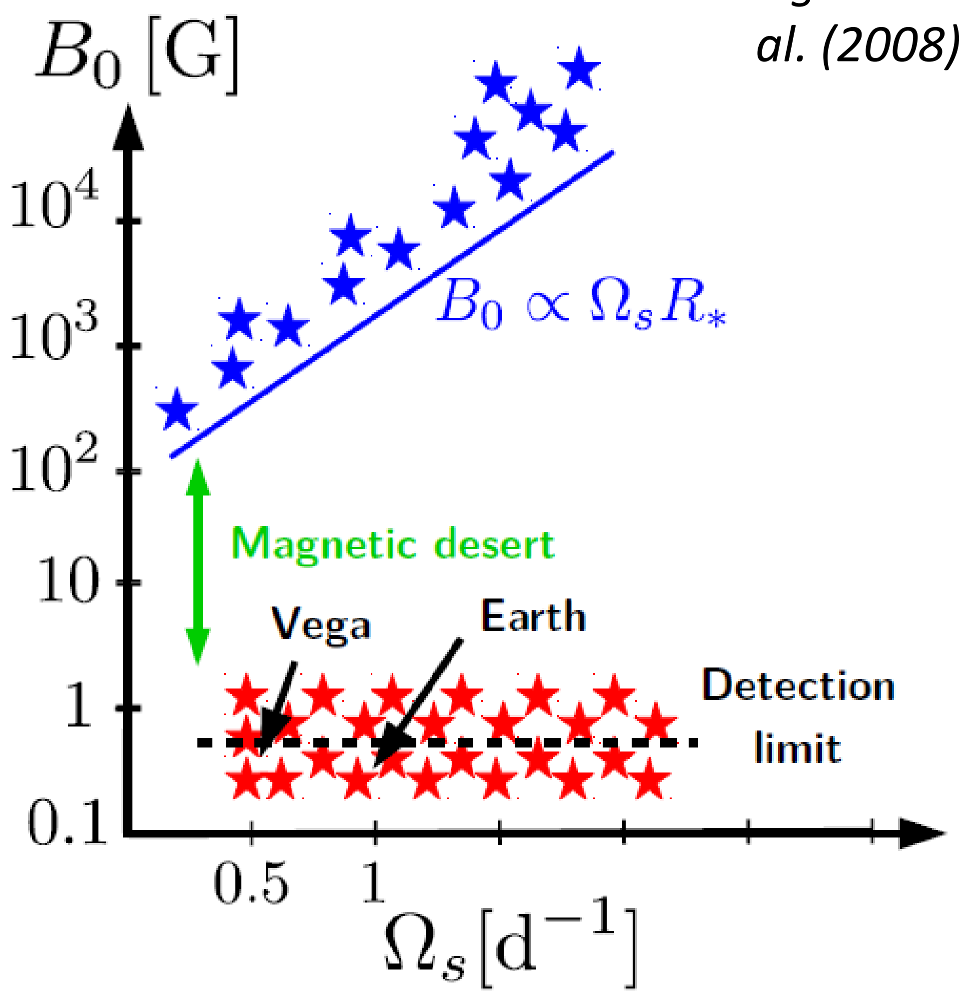
Strong fields B_0 (e.g. Ap stars)

- Typical **strength**: 10^2 - 10^4 G
- Often **static** fields
- Surface fields B_0 with a **fossil** origin

Ultra-weak fields B_0 (e.g. Vega)

- Typical **strength**: 0.1 - 1 G
- Often **dynamical** fields
- Surface fields B_0 generated by **dynamos**?

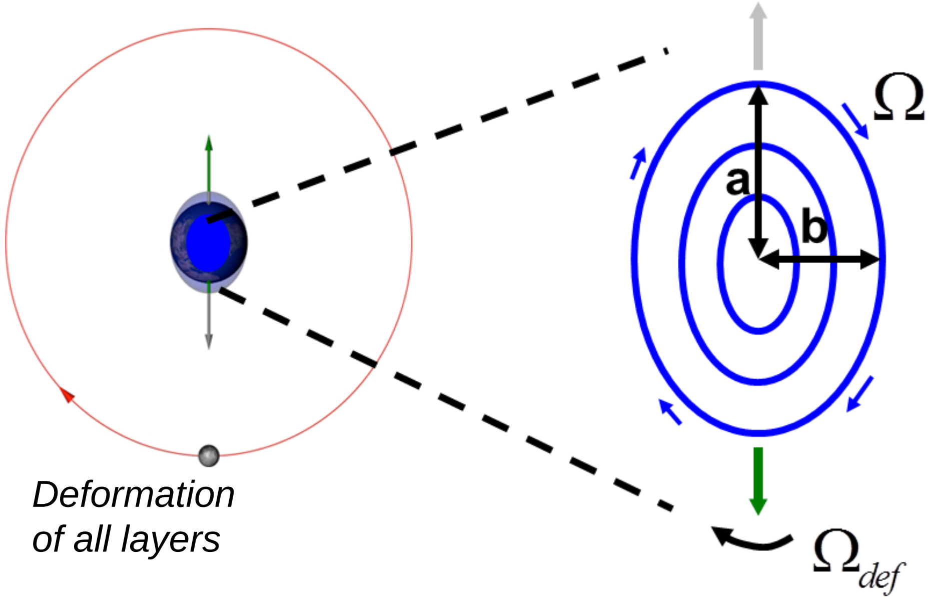
Lignières et al. (2008)



Tidal dynamos for Vega-like magnetism?

Tidal instability

Tides → Elliptical streamlines



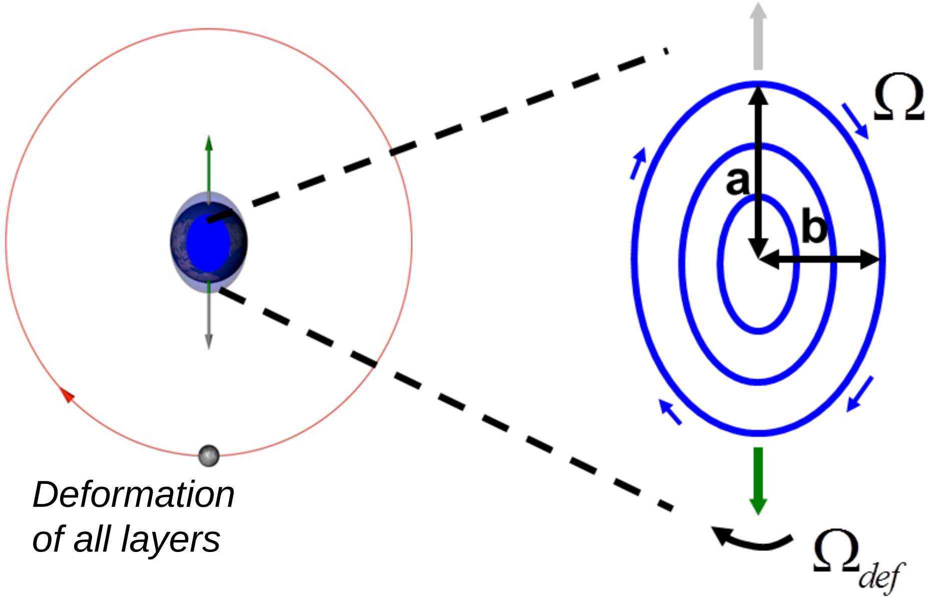
Tidal instability

Tides → Elliptical streamlines → Turbulent flows

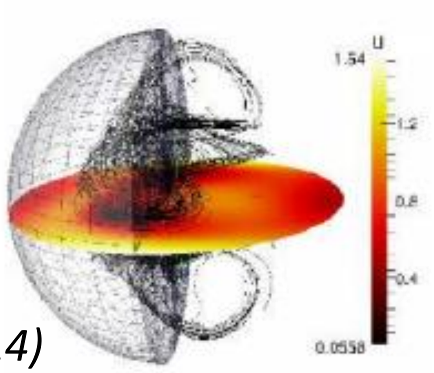
Elliptical or tidal instability!

?

Magnetic field



Deformation of all layers



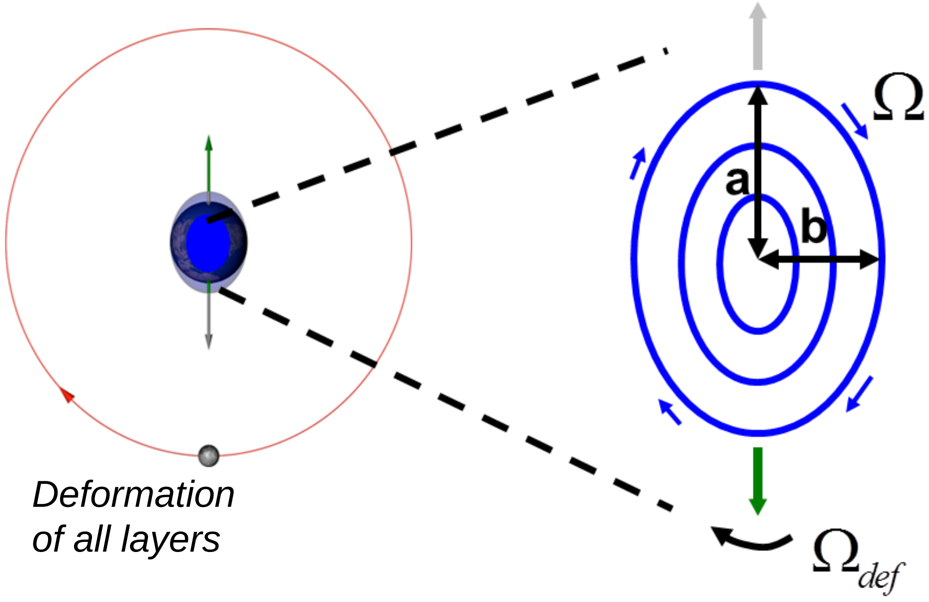
Cébron & Hollerbach (2014)

Tidal instability

Tides → Elliptical streamlines ⟶ **Turbulent flows**

Elliptical or tidal instability!

? ↓ **Magnetic field**



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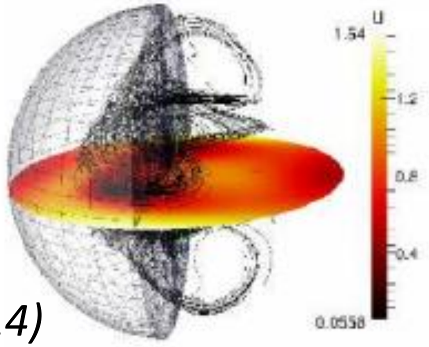

Cébron & Hollerbach (2014)

- **OK** for **neutral** or **convective** fluids (Cebron et al. 2010)
- But **tidally driven dynamos in stably stratified fluids?**

  → **radiative** stars, **lunar** core, **Encelade**, **Earth's** liquid core **stratified layer**, etc.

Stable stratification

- **(Dimensional) Brunt-Väisälä frequency N_0**

$$N_0^2 = -\alpha \nabla T \cdot \boldsymbol{g}$$

(α: Coeff. of thermal expansion, $\boldsymbol{g} = -\nabla\Phi$: gravity)

  - $N_0^2 > 0 \implies$ **stably stratified**,
  - **Internal** gravity waves.

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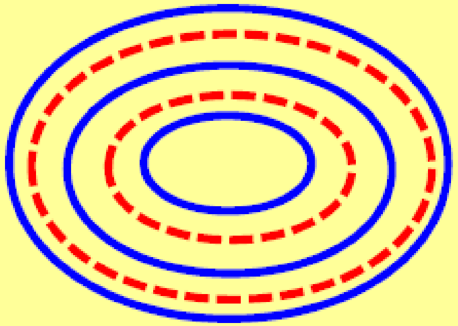
- ▸ $N_0^2 > 0 \Longrightarrow$ **stably stratified**,
- ▸ **Internal** gravity waves.

- **Growth rate elsewhere? Dynamo?**

- **Hypothesis**: - **Boussinesq** fluid
  - **Barotropic** & **linear** gravity

Barotropic



- No **baroclinic** instability,
- Growth rate **reduced** at the poles & equator.

Mathematical formulation

- Basic state: $\boldsymbol{U}_0, T_0, \boldsymbol{g}$,
- Scales: R_*, Ω_s^{-1}, $\Omega_s^2 R_* / (\alpha g_o)$ and $R_* \Omega_s \sqrt{\mu_0 \rho_*}$,
- **Dimensionless** numbers

$$Ek = \frac{\nu}{\Omega_s R_*^2} \leq 10^{-16}, \quad 10^{-6} \leq Pr = \frac{\nu}{\kappa} \leq 10^{-4}, \quad 10^{-8} \leq Pm = \frac{\nu}{\eta_m} \leq 10^{-4},$$

- Ellipsoidal container of equatorial ellipticity β_0.

Boussinesq equations of the perturbations $(\boldsymbol{u}, \theta, \boldsymbol{B})$ in the **inertial** frame

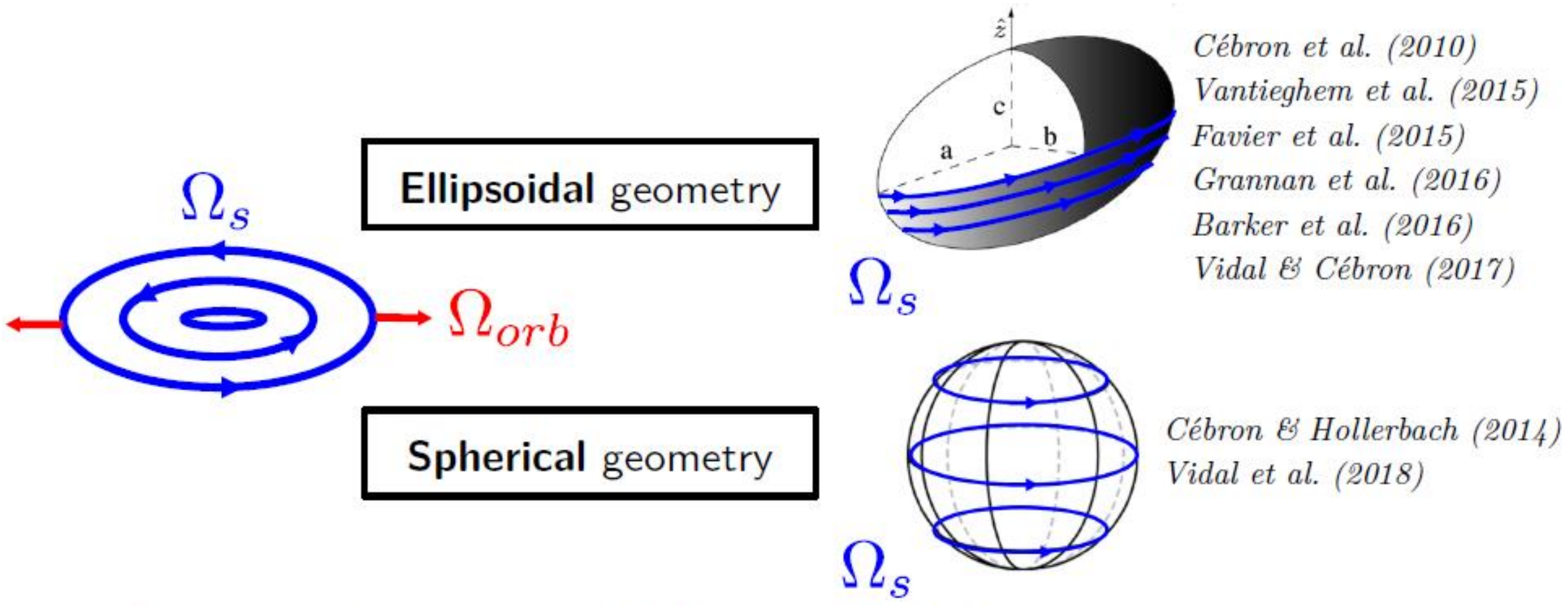
$$\frac{\partial \boldsymbol{u}}{\partial t} = -(\boldsymbol{u} \cdot \boldsymbol{\nabla})\, \boldsymbol{U}_0 - (\boldsymbol{U}_0 \cdot \boldsymbol{\nabla})\, \boldsymbol{u} - (\boldsymbol{u} \cdot \boldsymbol{\nabla})\, \boldsymbol{u} - \nabla p + Ek\, \nabla^2 \boldsymbol{u} - \theta\, \boldsymbol{g} + (\boldsymbol{\nabla} \times \boldsymbol{B}) \times \boldsymbol{B},$$

$$\frac{\partial \theta}{\partial t} = -(\boldsymbol{U}_0 \cdot \nabla)\, \theta - (\boldsymbol{u} \cdot \nabla)\, T_0 - (\boldsymbol{u} \cdot \nabla)\, \theta + \frac{Ek}{Pr} \nabla^2 \theta,$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{U}_0 \times \boldsymbol{B}) + \boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{B}) + \frac{Ek}{Pm} \nabla^2 \boldsymbol{B},$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = \boldsymbol{\nabla} \cdot \boldsymbol{B} = 0.$$

Numerical method



Spectral codes are more **efficient** & **reliable** for dynamo computations.

3 codes: COMSOL, YALES2, **NEK5000**

Various codes: **MAGIC**, ASH, **XSHELLS**, **PARODY**, **RAYLEIGH**

in red: open-source and freely available code

Numerical method

Cébron et al. (2010)
Vantieghem et al. (2015)
Favier et al. (2015)
Grannan et al. (2016)
Barker et al. (2016)
Vidal & Cébron (2017)

Cébron & Hollerbach (2014)
Vidal et al. (2018)

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World fastest spherical dynamo Boussinesq code
(benchmark of Matsui+16)

Numerical method: XSHELLS with basic states ***Schaeffer (2013, 2017)***

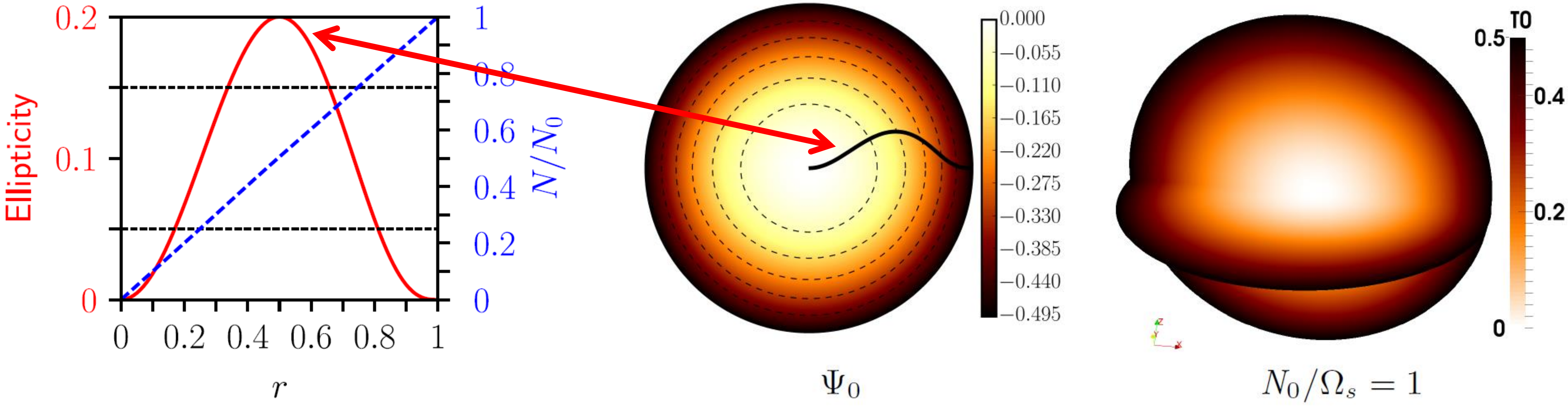
- **Open-source** code (https://bitbucket.org/nschaeff/xshells),
- Pseudo-spectral code with finite differences and spherical harmonics,
- Simulations **massively parallel** supercomputers (e.g. OCCIGEN): 2×10^6 h.c.

Radiative stars: idealized model *(Vidal et al. MNRAS, 2018)*

- Fixed $Ek = 10^{-4}$, $Pr = 1$ and $\Omega_0 = 0$,
- **Control** parameters

$$N_0/\Omega_s \text{ and max. tidal } \textbf{ellipticity } \epsilon \ll 1,$$

- **Basic** state: $\boldsymbol{U}_0(\Psi_0)$, $T_0(\Psi_0)$, $\boldsymbol{g}(\Psi_0)$,
- Barotropic state $(\boldsymbol{g} \times \nabla T_0 = \boldsymbol{0})$,
- BC: **Stress-free**, **fixed** temperature and electrically **insulating**.



Hydrodynamic instability *(Vidal et al. MNRAS, 2018)*

- $Ek = 10^{-4}, Pr = 1, \Omega_0 = 0,$
- **Onset** at $\epsilon_c = 0.054 \ (N_0/\Omega_s = 0)$

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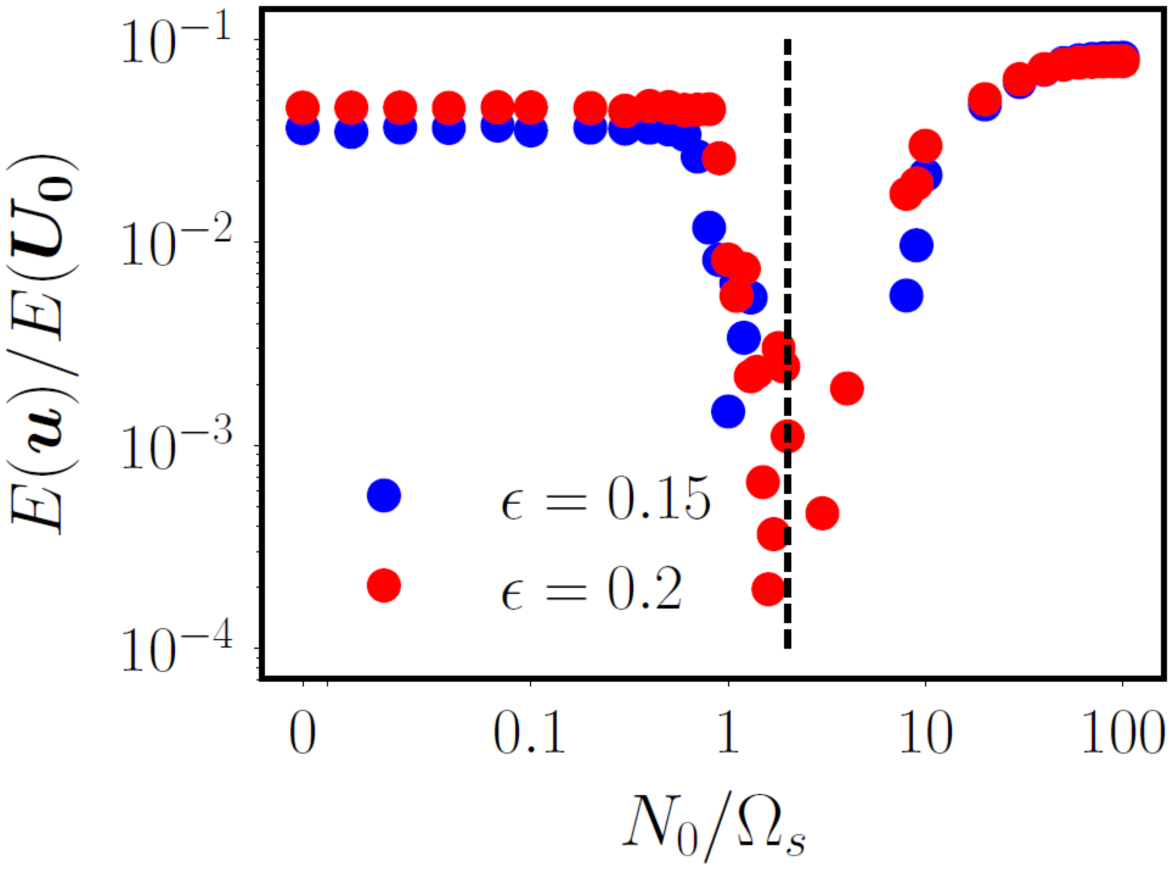

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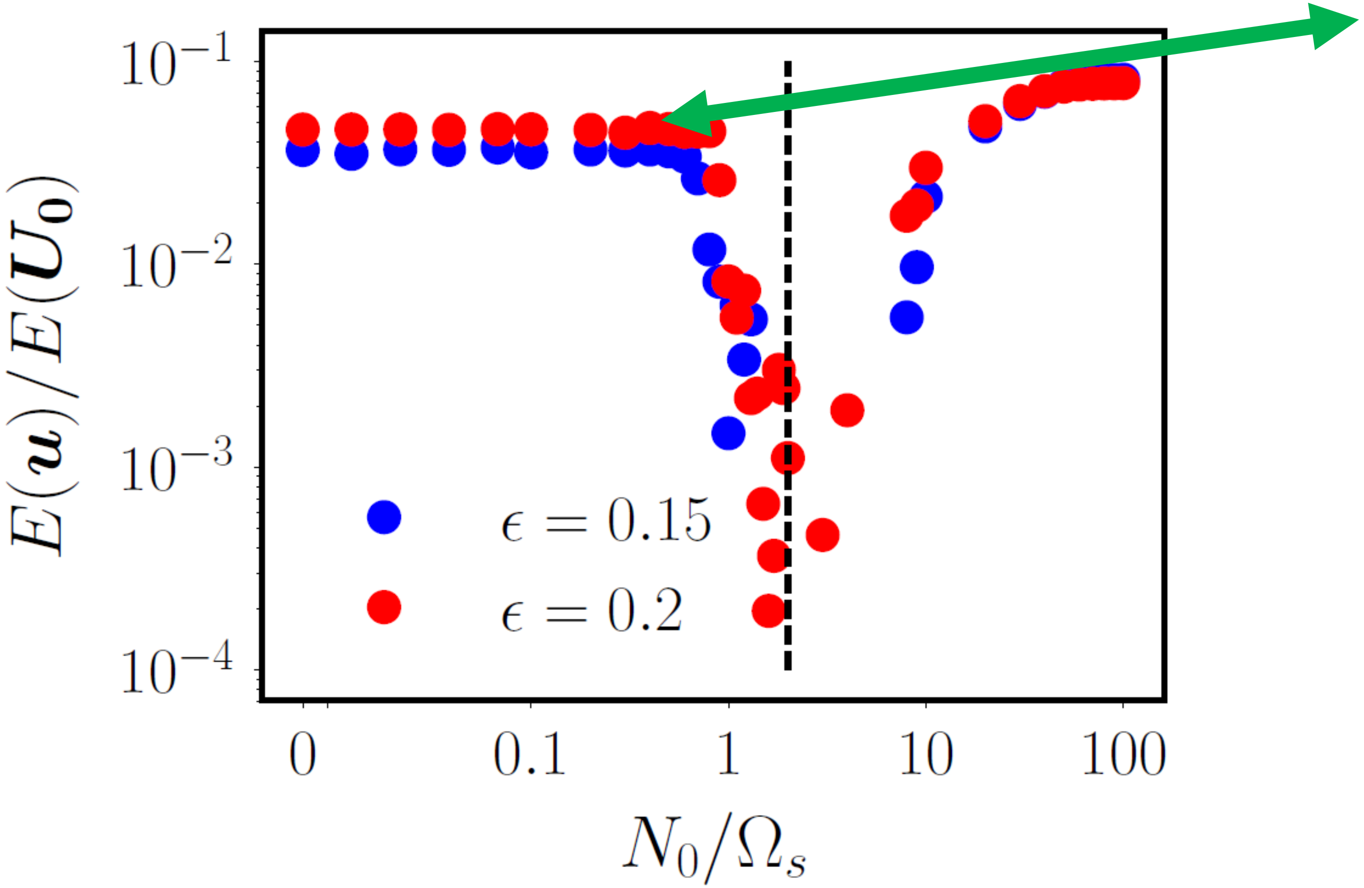
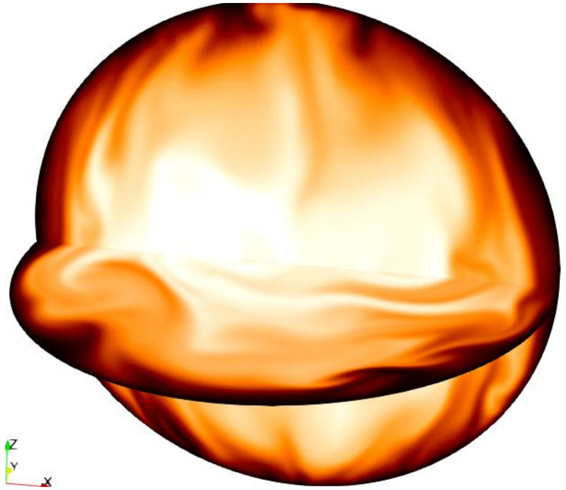


Total temperature



ε=0.2
$N_0/\Omega_s = 0.5$

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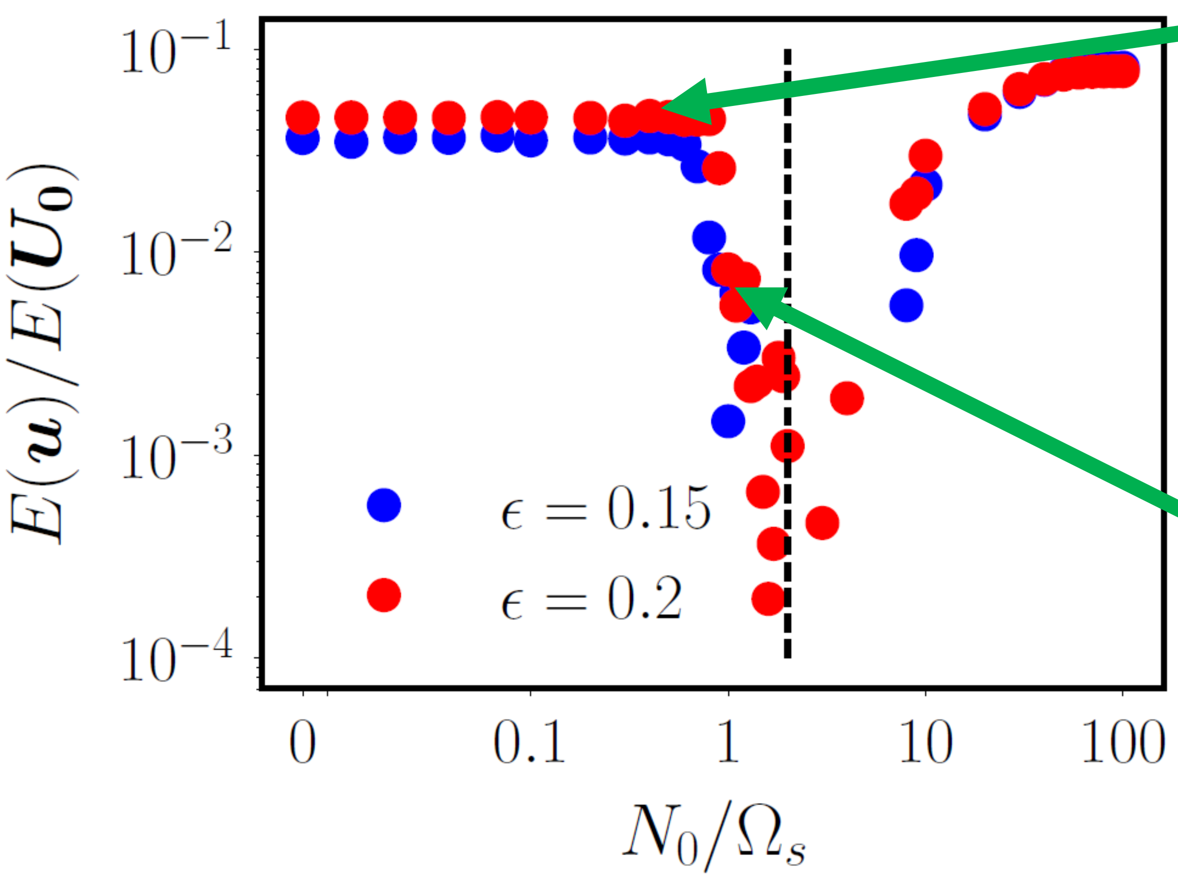

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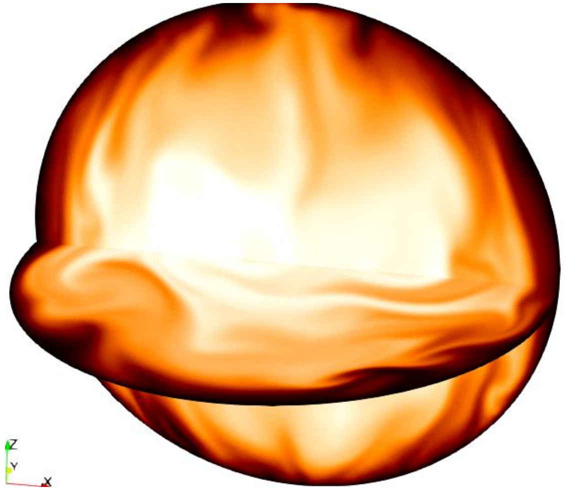
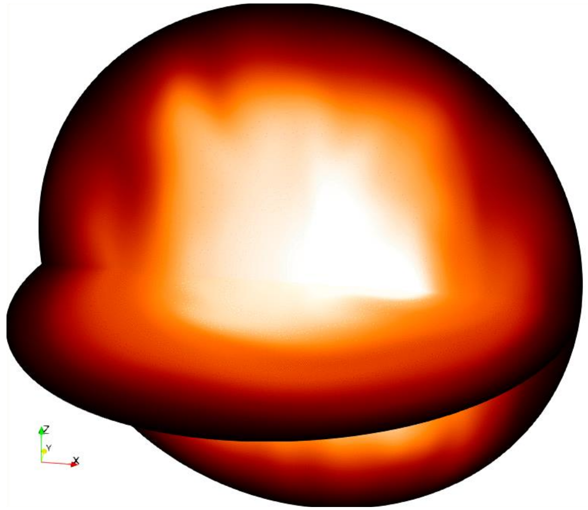


ε=0.2
N_0/Ω_s = 0.5



ε=0.2
N_0/Ω_s = 1

MHD simulations (Vidal et al., MNRAS, 2018)

- Basic flow $\boldsymbol{U}_0$ **not dynamo capable** (for **magnetic Prandtl** numbers $Pm \leq 5$),
- Integration over one **magnetic diffusive time**.

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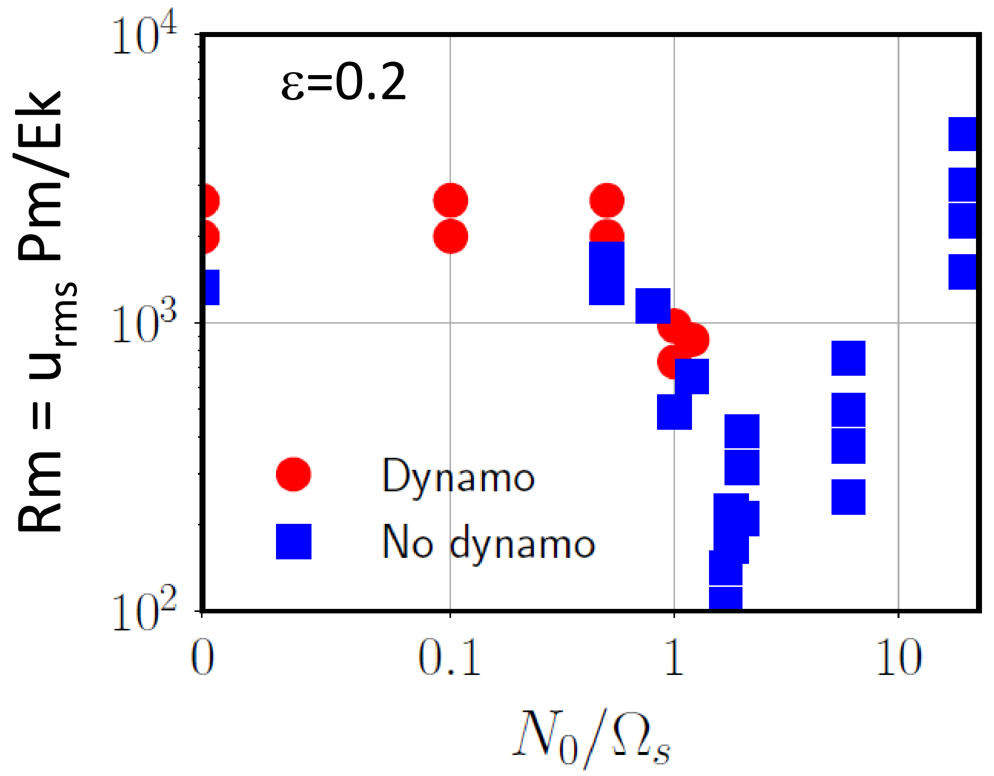

Dynamo for Rm>2000 & Pm~1

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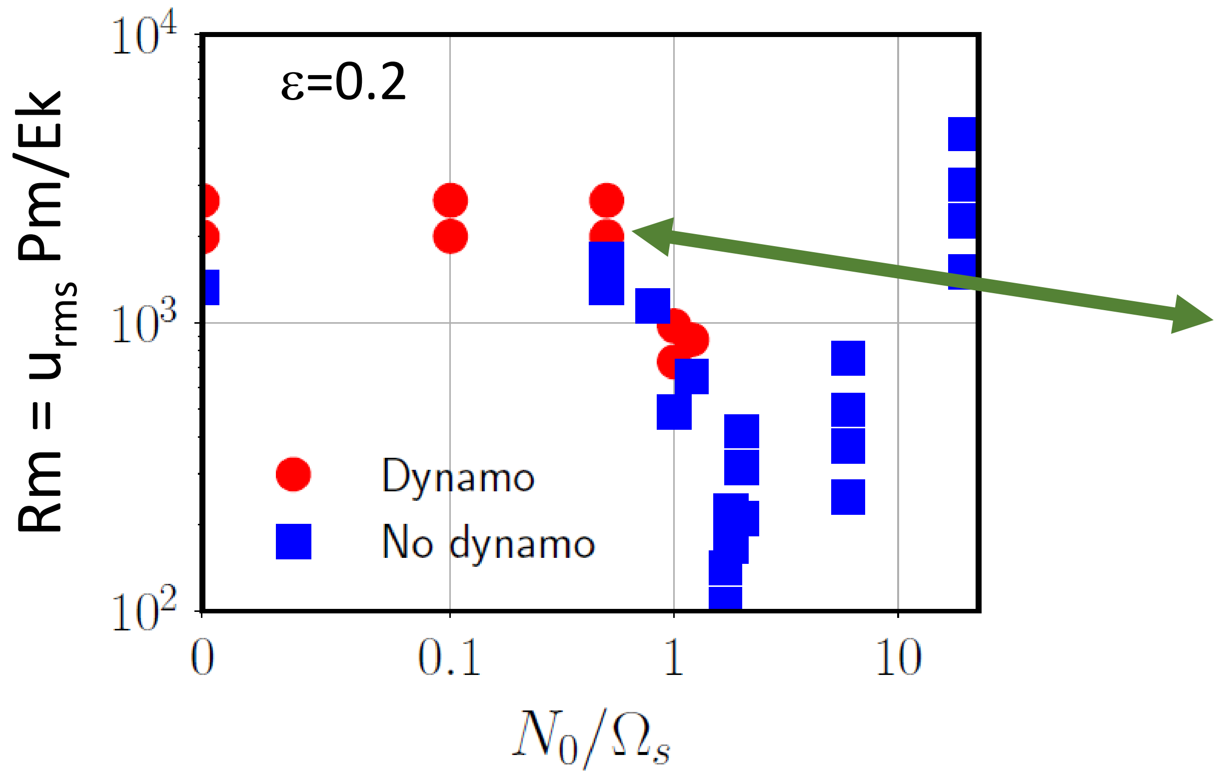
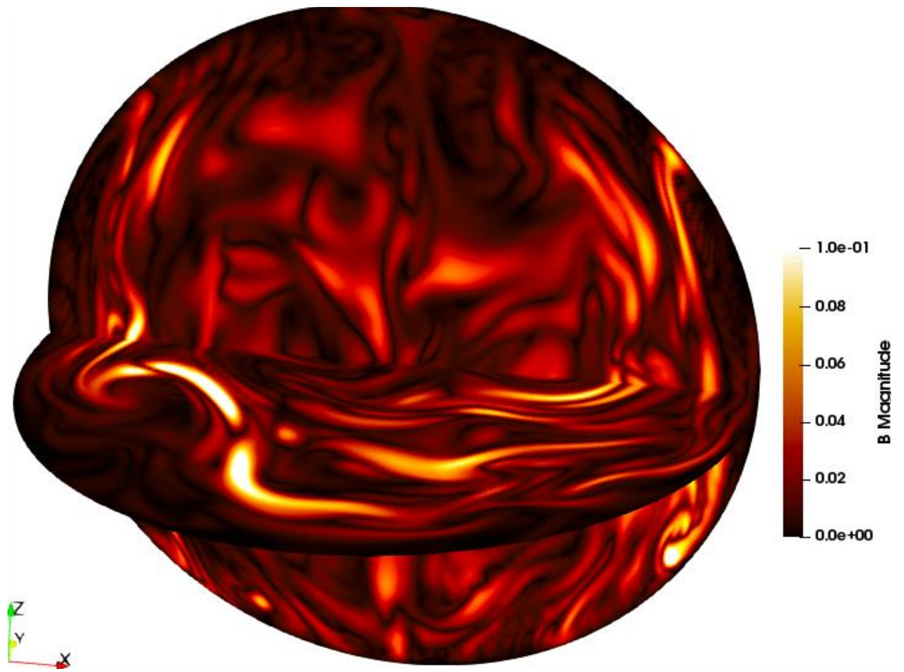
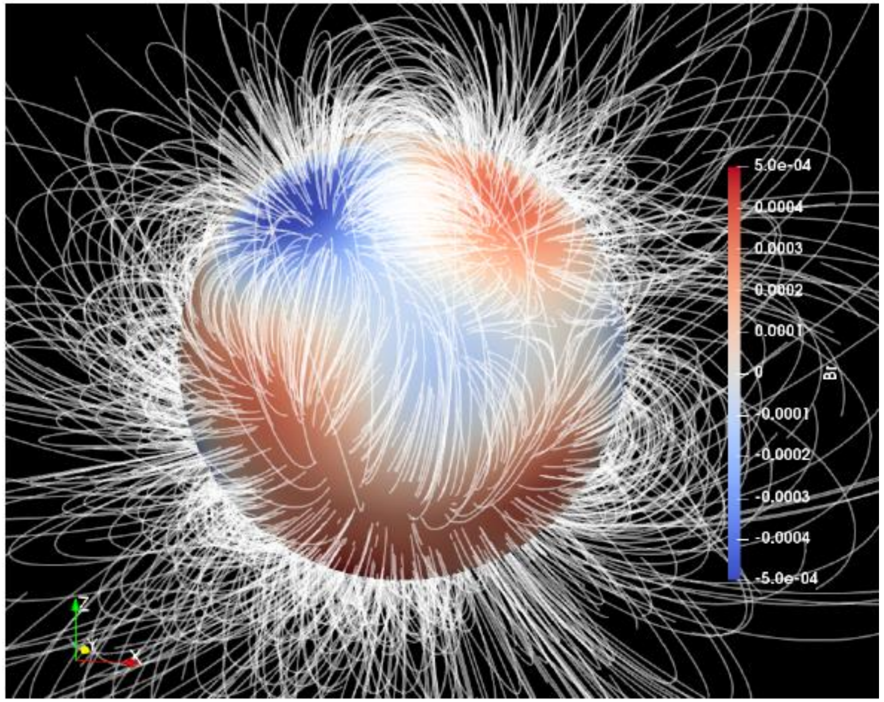




Dynamo for Rm>2000 & Pm>1

Saturated dynamo

Extrapolation (Vidal et al., MNRAS, 2018)



Large-scale surface field!

- Similar to convective scalings,
- Typical surface field **strength**

$$B_0 = \delta \frac{3}{2} \sqrt{\frac{3\mu_0}{4\pi}} \frac{R_*^{5/2}}{M_*^{1/2}} \Omega_s \frac{m}{D^3} \left|1 - \Omega_0\right|, \qquad 10^{-3} \leq \delta \leq 10^{-2}$$

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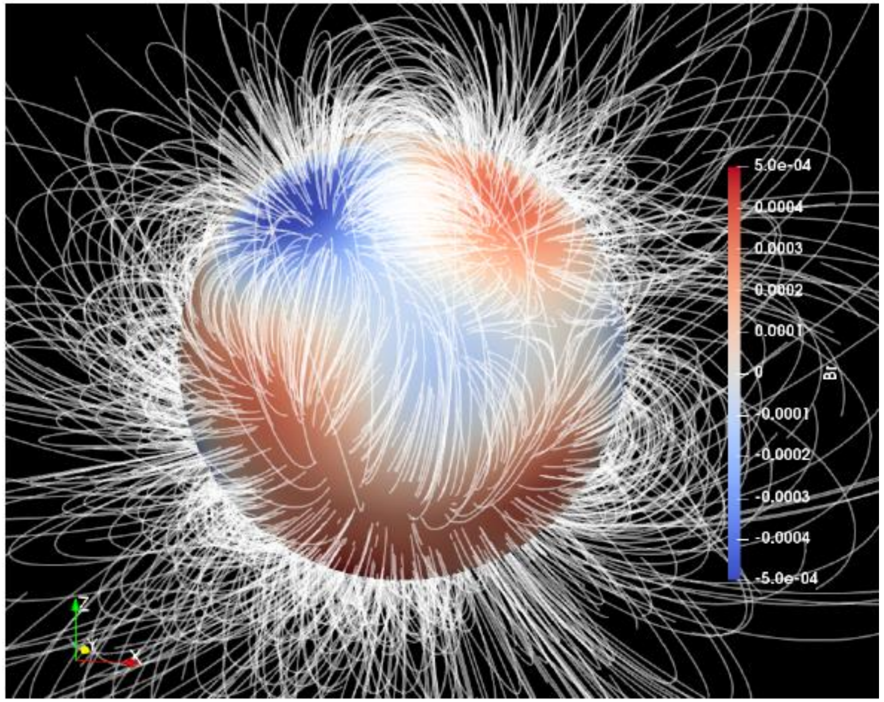

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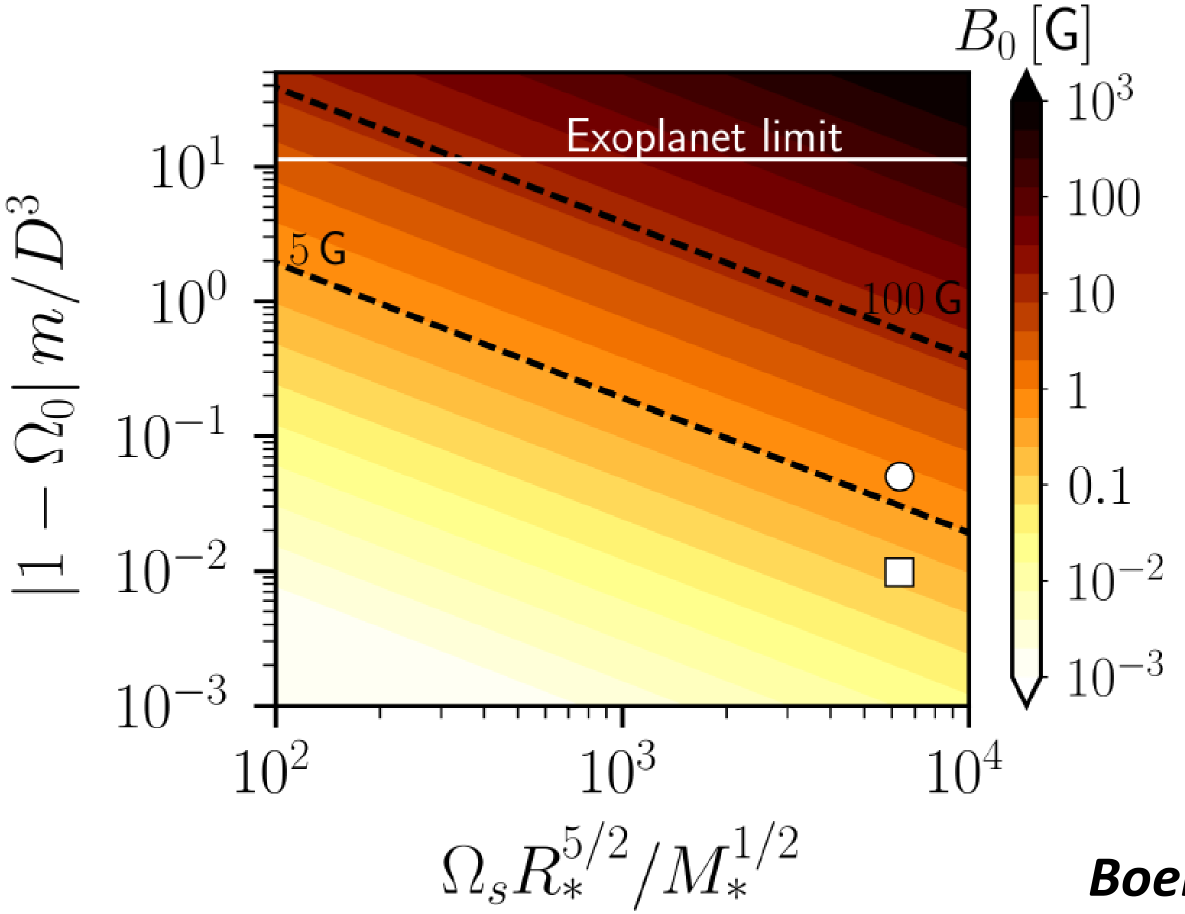


Vega

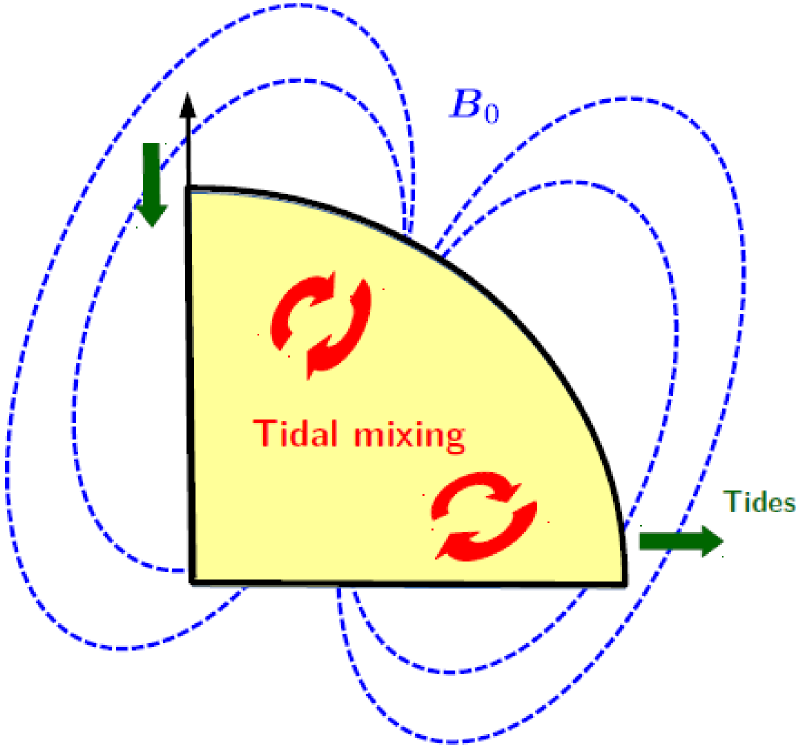
Measure:
0.6 ± 0.3 G

Theory:
1 - 1.5 G

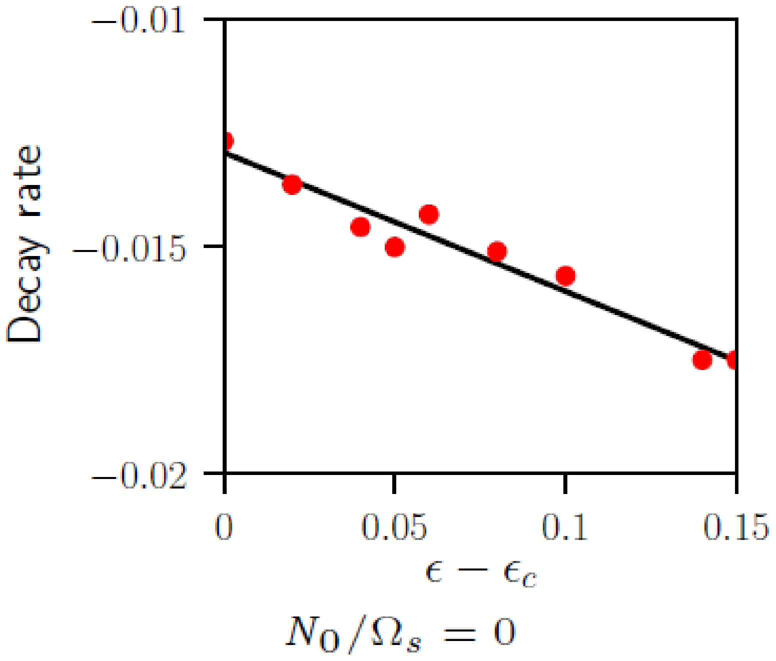
Boehm et al., 2015

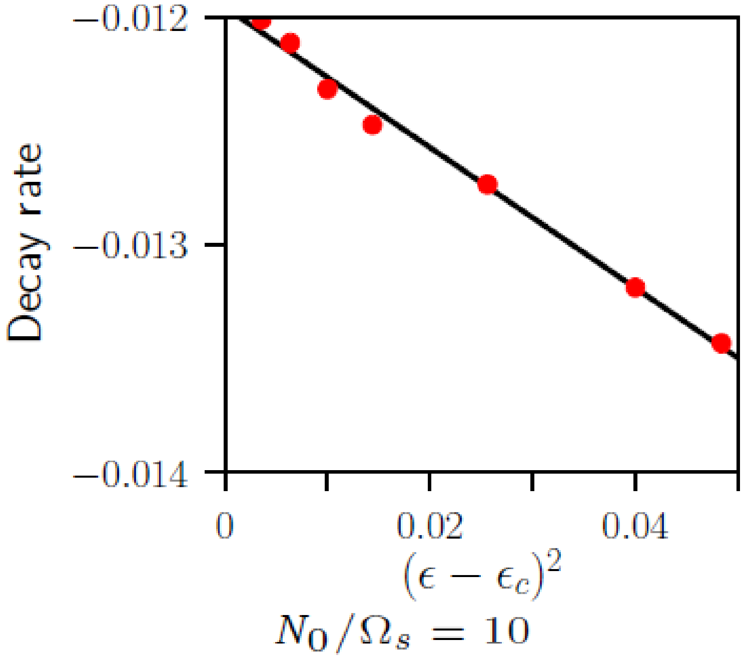
Perspective: tidal mixing of fossil fields

Ongoing *collaboration with Dr. Evelyne Alecian (IPAG) & Dr. Asif ud-Doula (Dunmore)*



Enhancing of Ohmic decay!





Conclusion

1) Tides can lead to mixing & dynamos in stratified fluids

- For moderate stratification (N/Ω <10 , see Vidal+18)
- For large enough tides

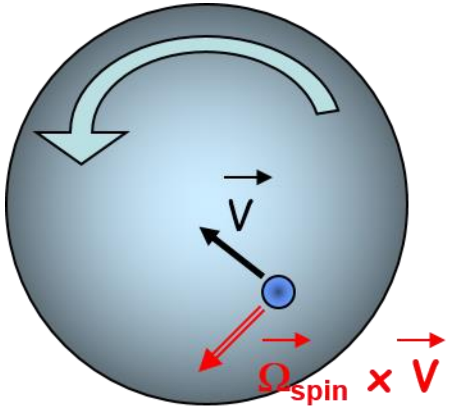
2) Tides can accelerate the fossile field (Ohmic) decay via the mixing

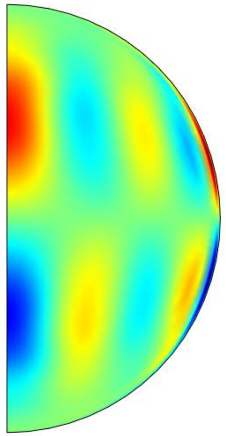
Thank you!

Inertial instabilities

Restoring force
(Coriolis, buoyancy, Lorentz force, etc.)

Waves, modes
(inertial, internal, Alfven, etc.)





Inertial instabilities

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Waves, modes
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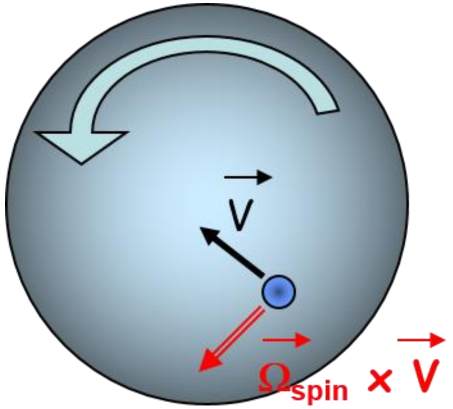
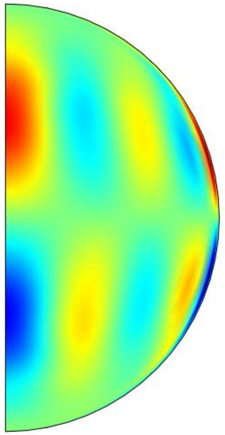
Periodic modulation

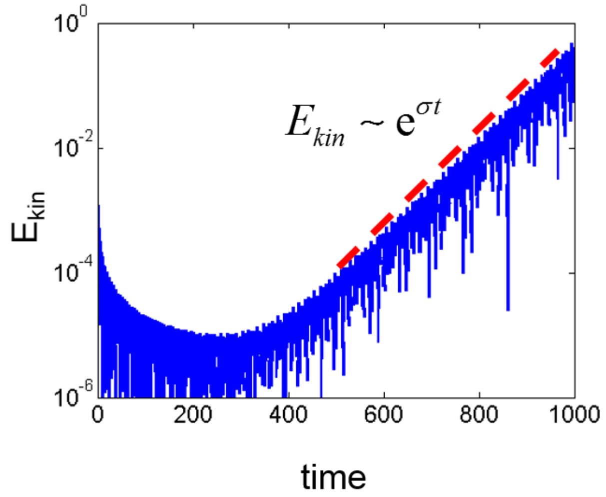
e.g. tides, precession, etc.

Parametric resonance
(tidal or precession instability, Faraday instability, etc.)





~ cos[m θ(t)]



Precession topographic **instability** ~ **Tidal instability** : similar mechanism
=> inertial instability (**parametric** resonance)