Dynamo action in rotating systems

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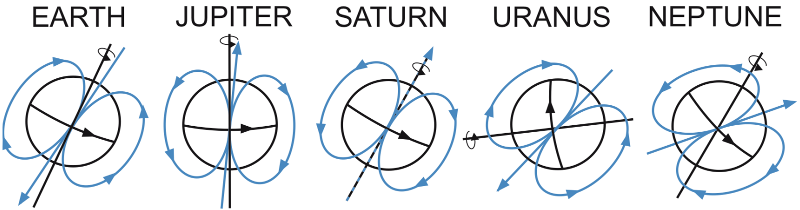
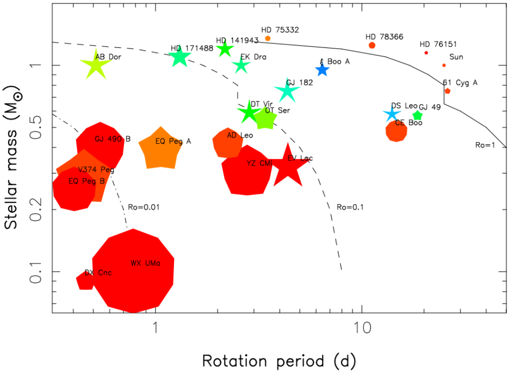
Different magnetospheric configurations



Axisymmetric ($m = 0$) and non-axisymmetric ($m = 1$) configurations

Ridley, V. A., Jovimagnetic secular variation, Doctoral thesis, 2013

Large-scale magnetic topologies of cool stars



Symbol size indicates relative magnetic energy densities, colour illustrates field configurations (blue/red for toroidal/poloidal fields respectively), shape depicts the degree of axisymmetry of the poloidal field component.

Donati, J.-F. & Landstreet, J. D., 2009, ARA&A, 47, 333

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MHD system

Dimensionless equations

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{1}{E}\nabla P + \frac{Ra}{E}\frac{r}{r_0}T\mathbf{e}_r - \frac{2}{E}\mathbf{e}_z\times\mathbf{v} + \nabla^2\mathbf{v} + \frac{1}{PmE}(\nabla\times\mathbf{B})\times\mathbf{B}, \quad (1)$$

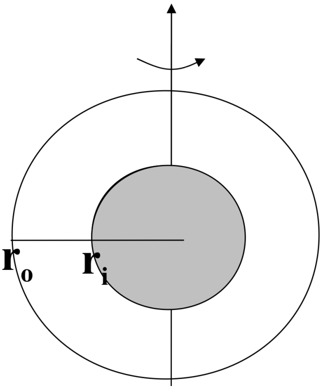
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}) + \frac{1}{Pm}\nabla^2\mathbf{B}, \quad (2)$$

$$\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T = \frac{1}{Pr}\nabla^2 T, \quad (3)$$

$$\nabla\cdot\mathbf{v} = 0, \qquad \nabla\cdot\mathbf{B} = 0. \quad (4)$$

$D = r_o - r_i$: unit of length, $\tau = D^2/\nu$: unit of time, $\sqrt{\rho\mu\eta\Omega}$: unit of B and ΔT: unit of temperature.

A simplified model



Set up

- A Boussinesq fluid in a rotating spherical shell with
  - ⋆ constant kinematic viscosity ν
  - ⋆ constant thermal diffusivity κ
  - ⋆ constant magnetic diffusivity η
- Convection is driven by an imposed temperature difference ΔT between the inner and outer sphere

Boundary conditions

- fixed temperature
- stress-free or no-slip for the flow
- insulating b. c. for the magnetic field

MHD system

Dimensionless equations

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} = Pm\Big[-\frac{1}{E}\nabla\frac{P'}{w^n} + \frac{Pm}{Pr}Ra\frac{S}{r^2}\mathbf{e}_r - \frac{2}{E}\mathbf{e}_z\times\mathbf{v} + \mathbf{F}^\nu + \frac{1}{E\,w^n}(\nabla\times\mathbf{B})\times\mathbf{B}\Big], \quad (5)$$

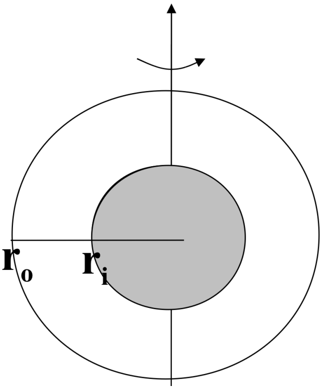
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}) + \nabla^2\mathbf{B}, \quad (6)$$

$$\frac{\partial S}{\partial t} + \mathbf{v}\cdot\nabla S = w^{-n-1}\frac{Pm}{Pr}\nabla\cdot\left(w^{n+1}\nabla S\right) + \frac{Di}{w}\left[E^{-1}w^{-n}(\nabla\times\mathbf{B})^2 + Q^\nu\right], \quad (7)$$

$$\nabla\cdot(w^n\mathbf{v}) = 0, \quad (8)$$

$$\nabla\cdot\mathbf{B} = 0. \quad (9)$$

A simplified model



Set up

- A perfect gas in a rotating spherical shell with
  - ⋆ constant kinematic viscosity $\nu = \mu/\rho$
  - ⋆ constant thermal diffusivity κ
  - ⋆ constant magnetic diffusivity η
- Convection is driven by an imposed entropy difference ΔS between the inner and outer sphere

Boundary conditions

- fixed entropy
- stress-free b. c. for the velocity field
- insulating b. c. for the magnetic field

Output parameters (1)

- kinetic and magnetic energy densities E_k and E_m
- Elsasser number $\Lambda = 2E_{\rm m}E/Pm$
- Rossby number $Ro = \sqrt{2E_k}E/Pm$ ("inertia/Coriolis")
- local Rossby number $Ro_\ell = Ro_c\,\ell_c/\pi$
  ℓ_c stands for the mean harmonic degree of the velocity component $\mathbf{v}_c$ from which the mean zonal flow has been subtracted

$$\ell_c = \sum_\ell \ell \frac{< (\mathbf{v}_c)_\ell \cdot (\mathbf{v}_c)_\ell >}{< \mathbf{v}_c \cdot \mathbf{v}_c >} . \tag{10}$$

- The topology of the field is characterized by the relative dipole field strength, $f_{\rm dip}$, defined as the time-average ratio on the outer shell boundary of the dipole field strength to the total field strength.

Dimensionless parameters in Boussinesq models

Order of magnitude for the simulations

Ra	$\frac{\alpha g \Delta T D}{\nu \Omega}$	$\leq 80 Ra_c$
Pm	ν/η	$0.20 \leq Pm \leq 12$
Pr	ν/κ	$0.3 - 3$
E	$\nu/(\Omega D^2)$	$3.10^{-4} \geq E \geq 10^{-5}$
χ	r_i/r_o	0.35 to 0.65
Λ	$B^2_{\text{rms}}/(\mu \varrho \eta \Omega)$	$0.05 \leq \Lambda \leq 45$
Ro	$U_{\text{rms}}/(\Omega L_c)$	$0.005 \leq Ro_\ell \leq 0.2$
Re	$(U_{\text{rms}} D)/\nu$	$10 \leq Re \leq 1100$
Rm	$(U_{\text{rms}} D)/\eta = Pm\,Re$	$40 \leq Rm \leq 1000$

Aim: Deduce systematic behaviours.

Systematic parameter studies (anelastic models)

Seven control parameters

Rayleigh number	Ra	$\frac{GMd\Delta S}{\nu\kappa c_p}$	$\mathcal{O}(10^6)$
magnetic Prandtl number	Pm	ν/η	$0.2 \leq Pm \leq 5$
Prandtl number	Pr	ν/κ	0.1 to 3
Ekman number	E	$\nu/(\Omega D^2)$	$10^{-3} \geq E \geq 10^{-5}$
aspect ratio	χ	r_i/r_o	0.35 to 0.65
polytropic index	n	$1/(\gamma-1)$	2
number of density scale heights	N_ϱ	$\ln(\varrho_i/\varrho_o)$	≤ 4

The anelastic version of PaRoDy reproduces the anelastic dynamo benchmark.

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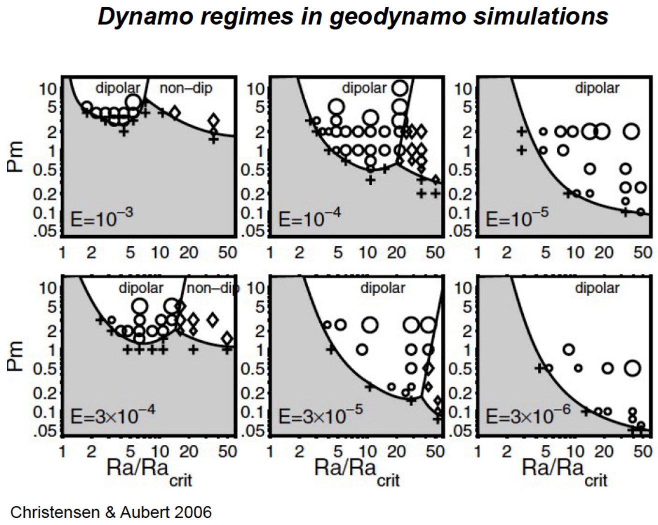
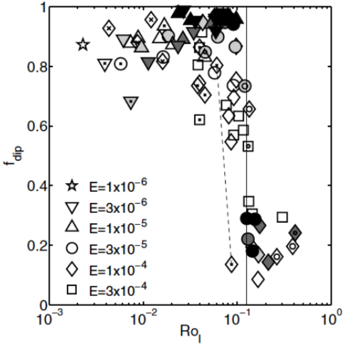
Extensive parameter survey of geodynamo models



In geodynamo models

Two distinct regimes in geodynamo simulations



Christensen & Aubert, 2006

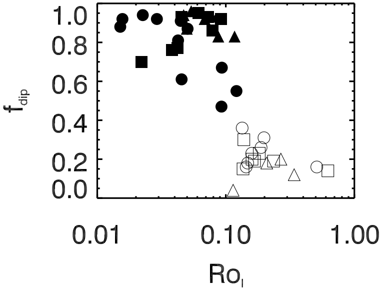
$$\bar{\ell}_u = \frac{\sum \ell \langle \mathbf{u}_\ell \cdot \mathbf{u}_\ell \rangle}{2E_{\rm kin}},$$

$$Ro_\ell = Ro\frac{\bar{\ell}_u}{\pi}.$$

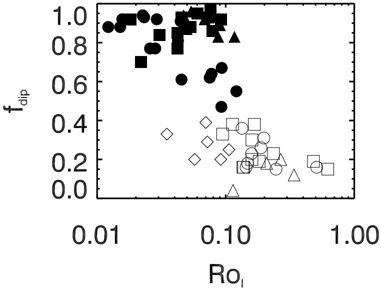
Dipole field strenght f_{dip}: time-average ratio on the outer shell boudary of the mean dipole field strength to the field strength in harmonic degrees l=1-12.

Existence of two dynamo branches

dipolar dynamo	multipolar dynamo
large scale field	frequent polarity reversals, oscillatory solutions
kinematically stable	kinematically unstable
mainly axisymmetric	mainly non axisymmetric



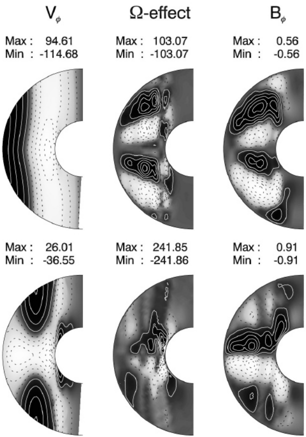
no-slip b. c.



stress-free b. c.

Schrinner, M., Petitdemange, L., & Dormy, E. 2012, ApJ, 752, 121

Bistability



Bistability
SF/SF B.C.

multipolar field branch

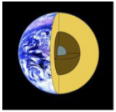
Oscillatory dynamos

Dipolar field branch

Dipolar and stationary

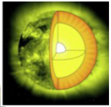
Schrinner, Petitdemange, Dormy (2012) Ap.

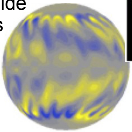
Dynamo regimes in Boussinesq and Anelastic models



De la géodynamo au magnétisme stellaire

Etude de la topologie magnétique à l'aide de simulations numériques directes



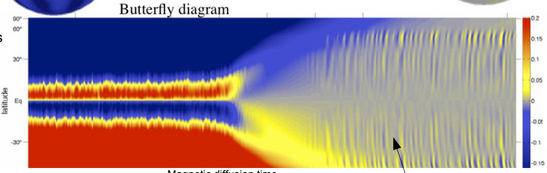


Effondrement de la composante Dipolaire du champ magnétique initialement dominante si $Ro_\ell > 0.1$

Résultats pertinents dans les modèles Boussinesq $\rho=\rho_0$ et anélastique :

$$N_\rho = \ln\left(\frac{\rho(r=r_i)}{\rho(r=r_o)}\right)$$

$$\vec{\nabla}\cdot(\rho(r)\vec{v}) = 0$$

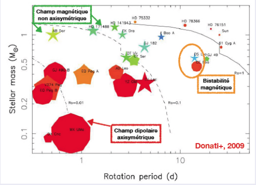


Nombre de Rossby local

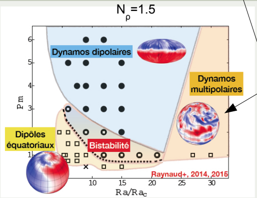
$$Ro_\ell = Ro\frac{L}{\ell_c} = \frac{u_{rms}}{\Omega L}\frac{L}{\ell_c}$$

ℓ_c : size of convection cells

Contexte observationnel



Contexte théorique



Nos études de paramètres, permettent de comprendre la génération des champs dans les zones convectives
Ex : la rotation différentielle joue un rôle important pour les dynamos oscillantes.

Schrinner et al (2012), ApJ.
Schrinner et al (2014) A&A
Raynaud et al (2014) A&A
Raynaud et al (2015) MNRAS
Petitdemange (2018) PEPI
Raynaud et al (2018) A&A

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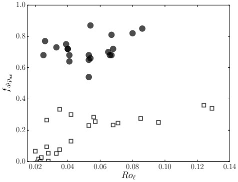
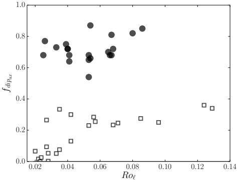
Role of inertia: local Rossby number depends on r

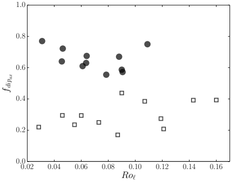
$N_\varrho = 0.5$



$N_\varrho = 1.5$

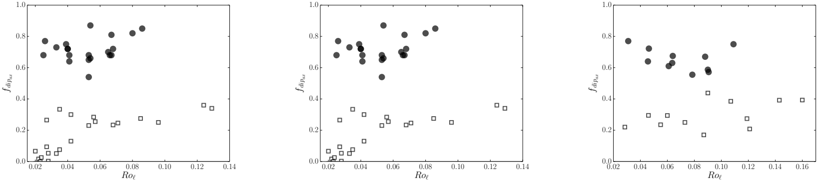


$N_\varrho = 2.0$

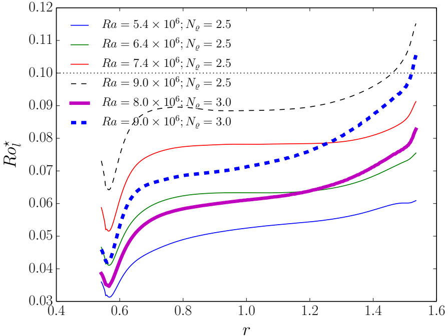


Role of inertia: local Rossby number depends on r

$N_\varrho = 0.5$ $N_\varrho = 1.5$ $N_\varrho = 2.0$

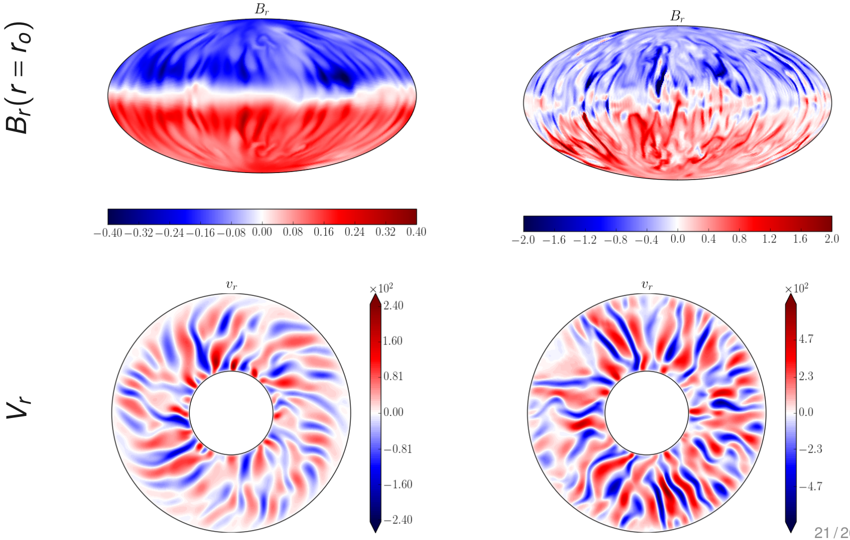


Local Rossby number as a function of radius for dipolar (solid lines) and multipolar (dashed lines) dynamos at ($N_\varrho = 2.5, Pm = 2$) (thin lines) and ($N_\varrho = 3, Pm = 4$) (thick lines).



Snapshots

$N_\varrho = 1.5, Pm = 0.75, Ra/Ra_c = 5$ | $N_\varrho = 2.5, Pm = 2, Ra/Ra_c = 3.4$



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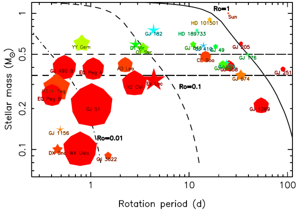
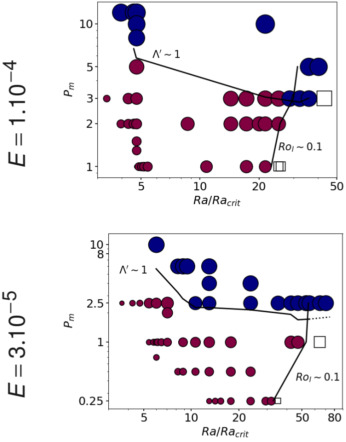
Strong/weak field dynamos in geoynamo simulations

Observations: Moutou *et al* 2017



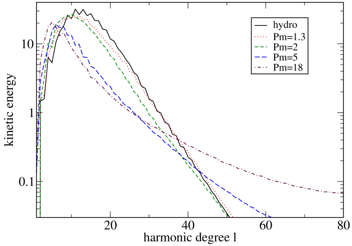
Dipolar dynamos with $Ro_\ell > 0.1$ in the strong field regime (high Pm as $Rm = Re\,Pm$.

Menu, Petitdemange, Galtier in prep



Strong field dynamos in geodynamo simulations when $\Lambda' > 1$ (Dormy 2016 *JFM*).

The magnetic fields strongly affects the convective length scale



Geodynamo simulations with the parameters: $E = 10^{-4}$, $Ra/Ra_C \approx 6$ and $Pr = 1$ (Petitdemange (2018) *PEPI*).

The magnetic field can strongly affect heat transfer and the velocity field. In strong field dynamos, the size of convection cells is almost the radius/gap of the convective zone.

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Conclusion and perspectives

- Varying dimensionless parameters of the aspect ratio affects Ro_ℓ.
- Dipolar fields cannot be maintained when Ro_ℓ if the field strength is weak ($\Lambda' < 1$).
- Dipolar fields are relevant $\forall N_\varrho$ but $Ro_l(r)$
- Existence of a bistable regime in Boussinesq/Anelastic models: oscillatory dynamos are $\alpha\Omega$-dynamos and dipole-dominated ones are α^2.
- Oscillatory dynamos : Parker waves

In progress:

- Strong field dynamos in anelastic models.
- Influence of Pr.
- Effect of the magnetic field on the gravity darkening.
- ...