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ÉCOLE NORMALE SUPÉRIEURE

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Why is the IMF so universal ?
An origin of the characteristic mass of stars

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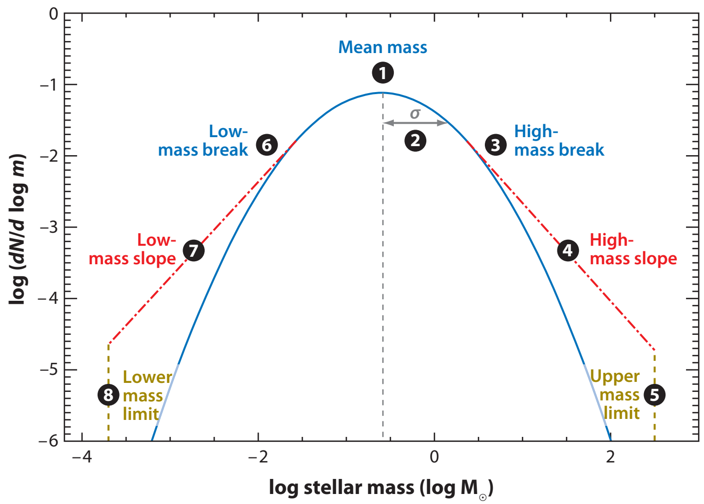
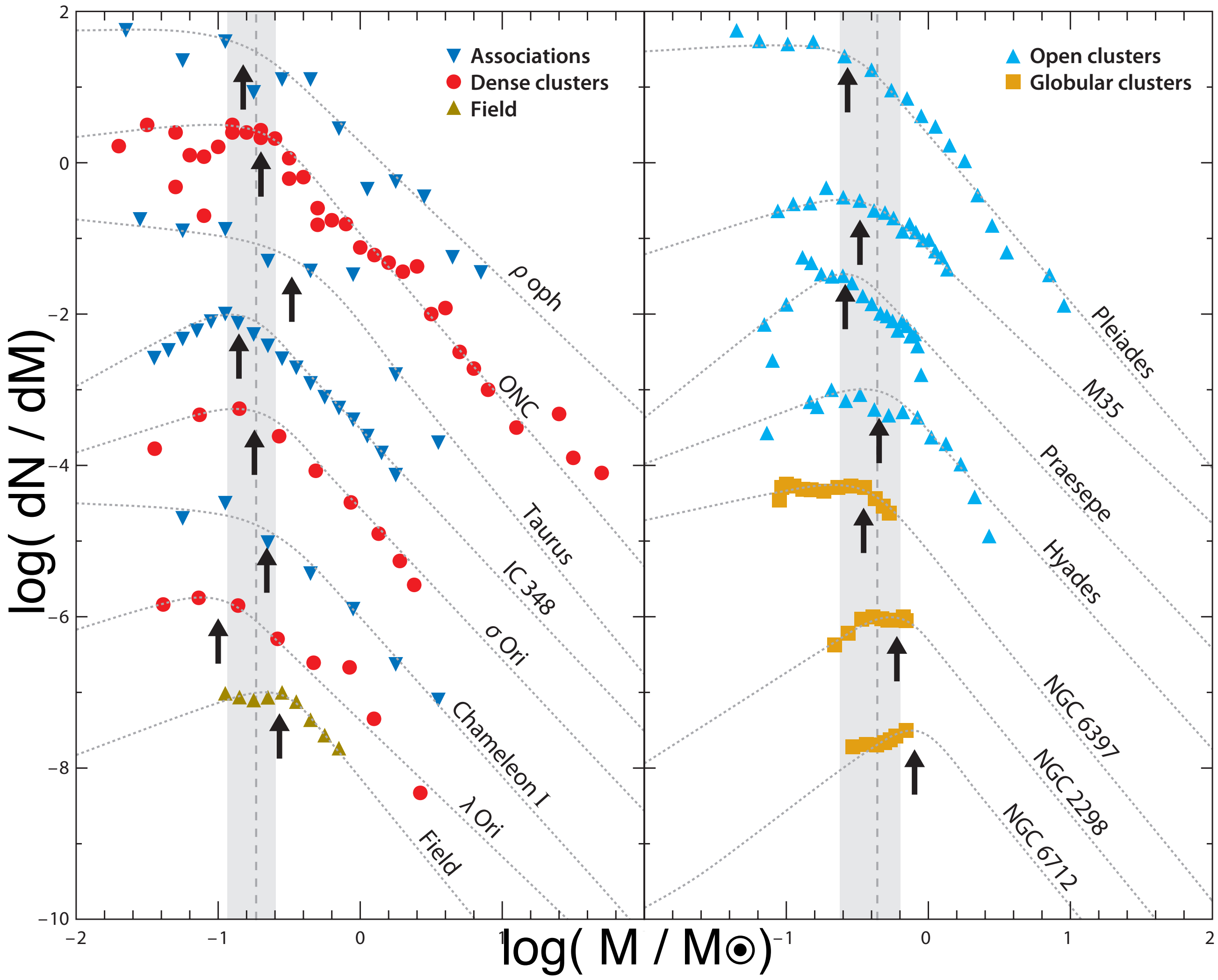
Thanks to: Gilles Chabrier

The Initial Mass Function

(Salpeter 1955, Kroupa 2002, Chabrier 2003, Hillenbrand 2004, Moraux+2007, Bastien+2010, Offner+2014)





IMF seems pretty universal
Why ?

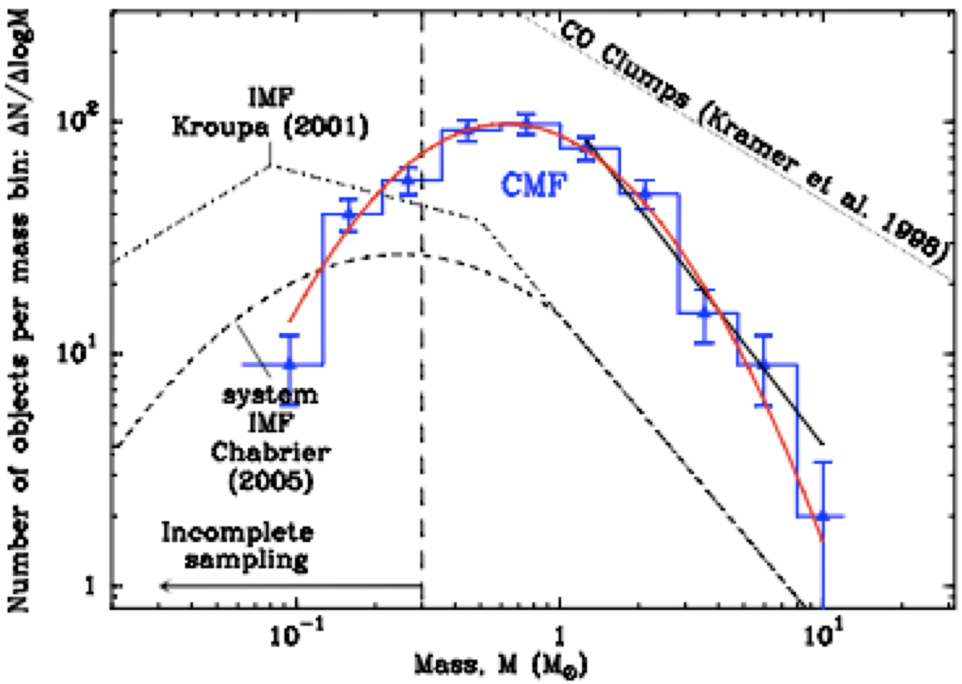
Bastien+2010

The Core Mass Function

(cores are progenitors of stars and likely set their mass)

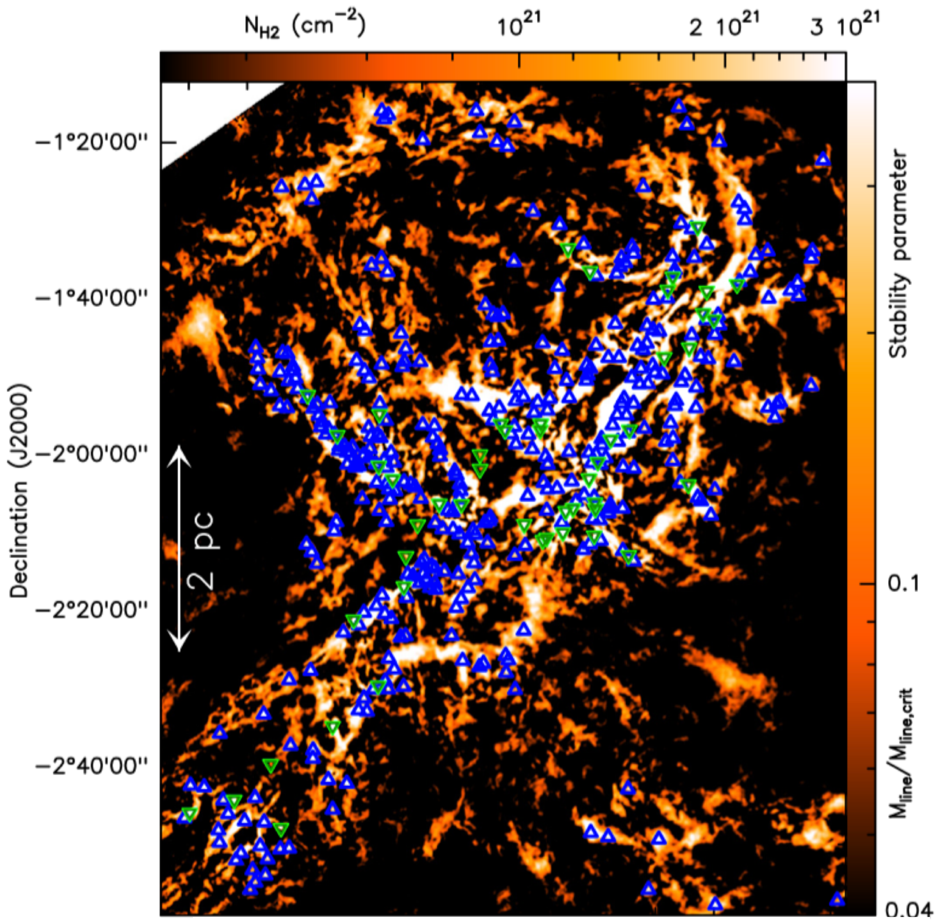
(Motte et al. 1998, Testi & Sargent 1998, Alves et al. 2007, Johnstone et al. 2002, Enoch et al. 2008, Simpson et al. 2008, André et al. 2010, Konyves et al. 2010, 2015)

CMF in Aquila molecular cloud



Is the Core mass function (CMF) at the origin of the initial mass function (IMF) ?

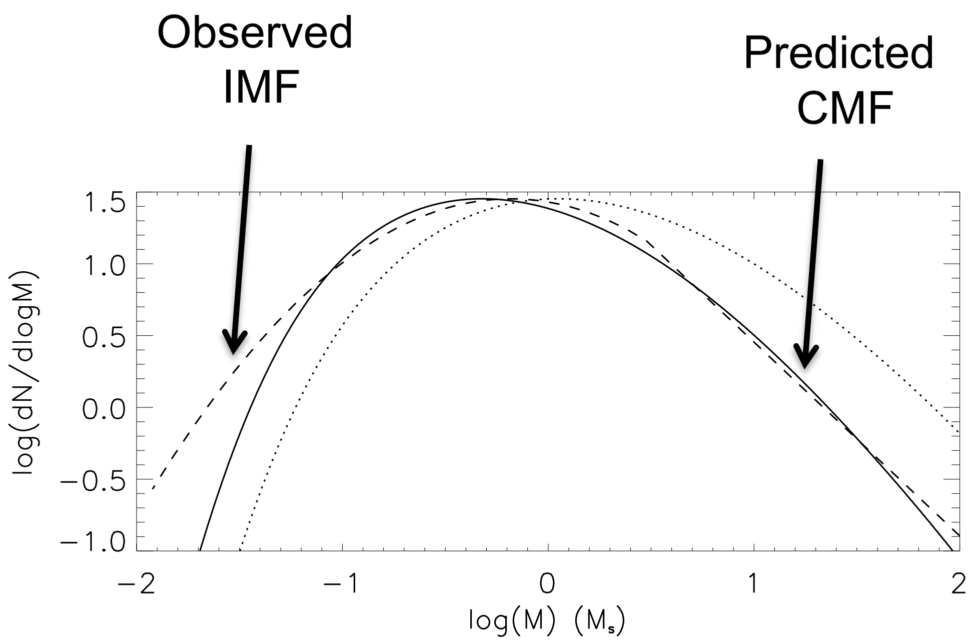
Core distribution from Herschel



Konyves, André et al. 2015

Many works have attempted to get the Core Mass Function

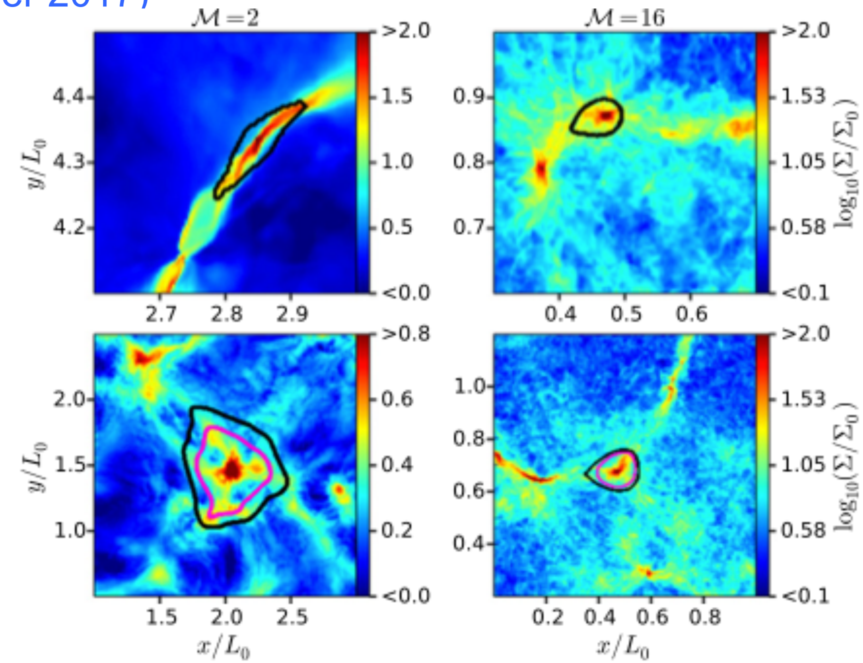
(e.g. Klessen 2000, Klessen & Burkert 2001, Padoan+2007, H& Chabrier 2008, Gong & Ostriker 2011, 2013, 2015, Hopkins 2013, Chen & Ostriker 2014, Lee, H & Chabrier 2017)

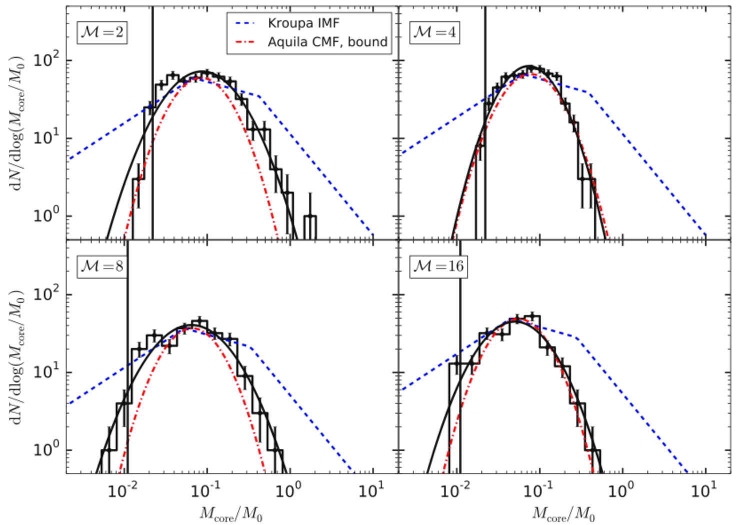


"Press & Schecter" type of analysis

Salpeter = gravity+turbulent support

H & Chabrier 2008





Gong & Ostriker 2011

Problem: results depend on initial conditions

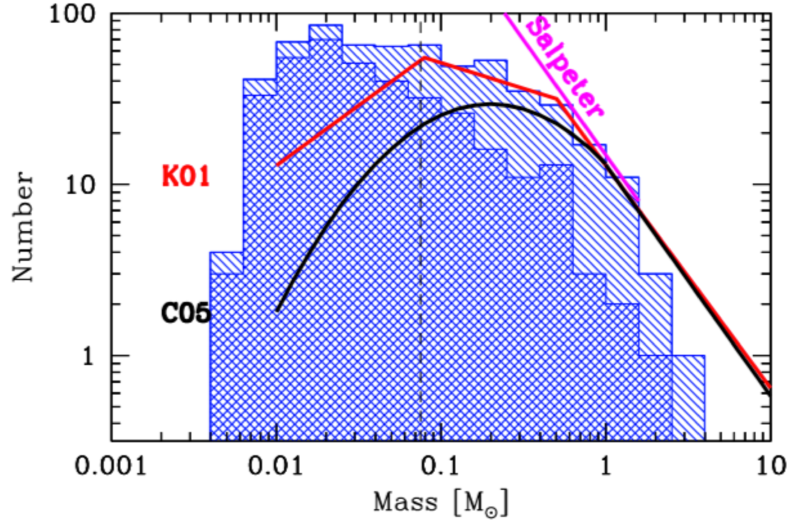
Vary with mean density for example =>

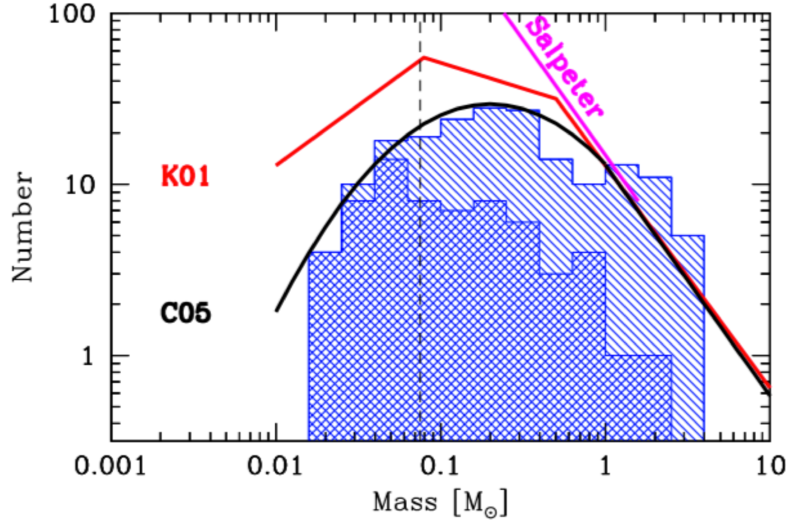
difficult to explain the universality of the IMF

Getting the IMF/sink-MF from collapsing molecular clouds

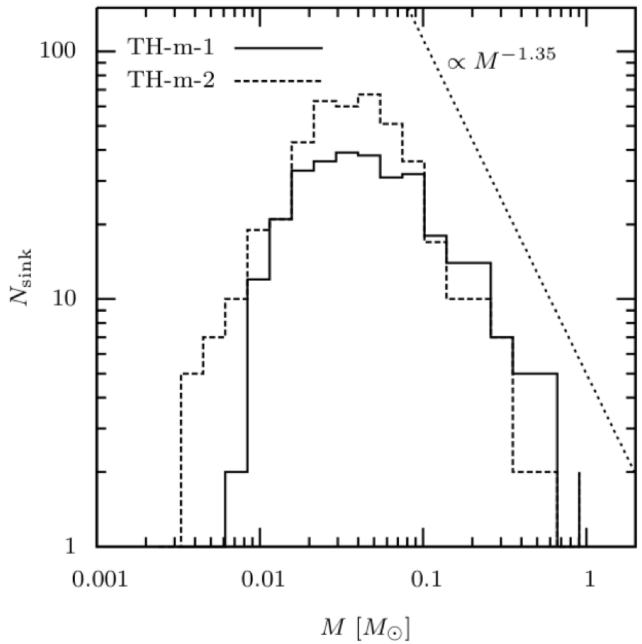
(Bate 2003, 2012…, Japsen+2005, Bonnell+2011, Krumholz+2012, Girichidis+2011, Ballesteros-Paredes+2015)

Bate 2012

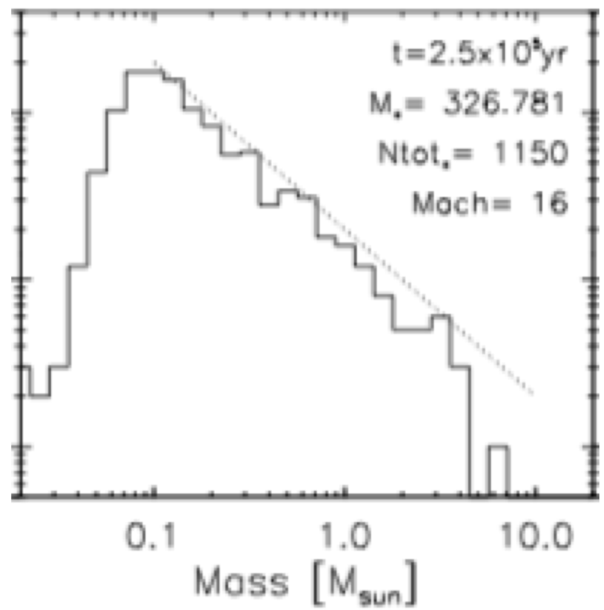




Girichidis+2011



Ballesteros-Paredes+2012



Performing a series of numerical experiments to investigate in a systematic way what are the important dependence of the mass spectrum

-RAMSES code is used (Teyssier 2002, Fromang+2006)

-1000 Ms clouds

-shallow density profile initially

-"turbulence" added (random phase) but the cloud is relaxed before gravity is switched on

-equation of state to mimic dust opacity at high temperature $T=T_0 (1 + (n/n_{ad}))^{\gamma}$
=> Isothermal to adiabatic transition

-sink particles being used (Bleuler & Teyssier 2013). Density threshold 10^{10-11} cm^{-3}

-numerical resolution goes from 17 to 2 AU, numerical convergence is checked

Density, turbulence, magnetic field, equation of state and numerical resolution are all varied.

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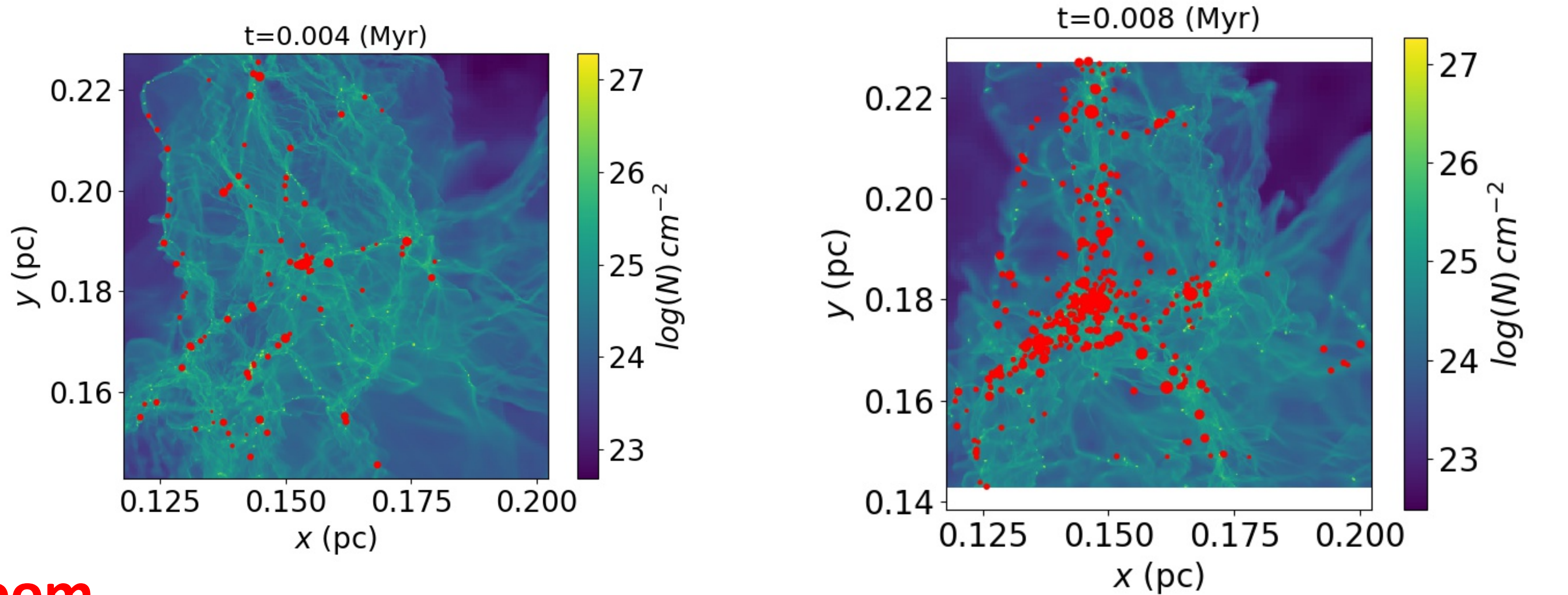
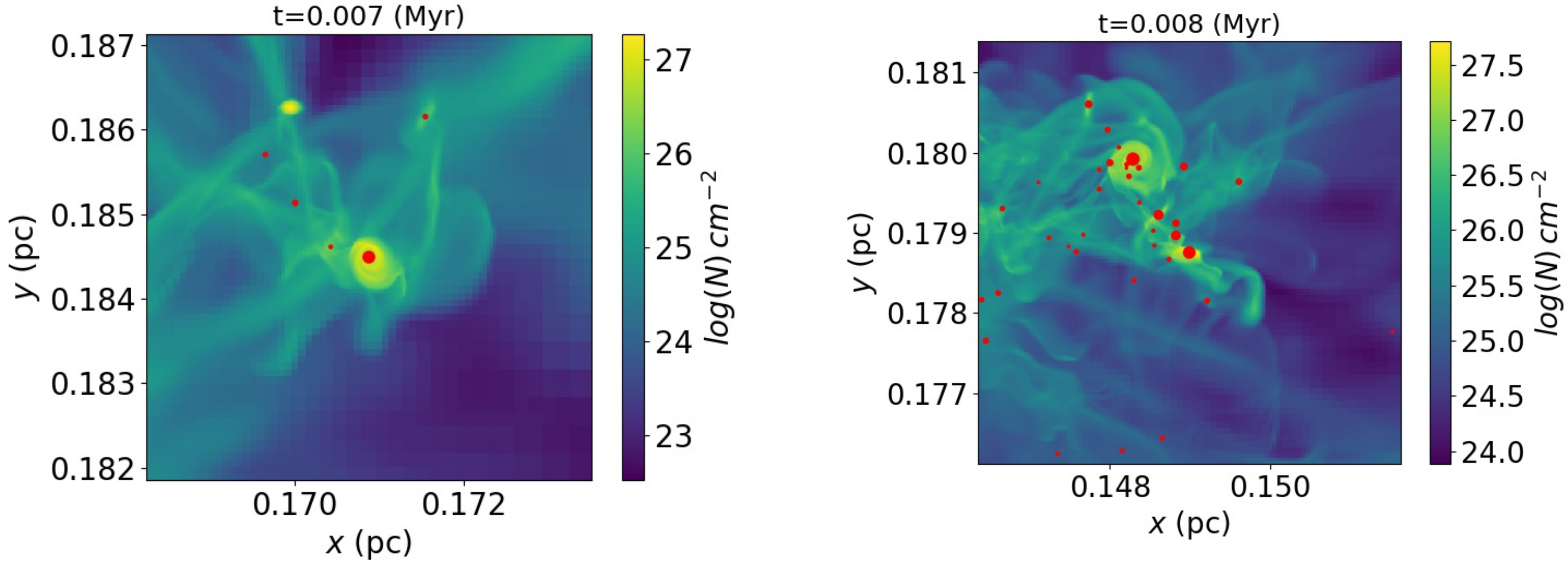
Some examples of column density and sinks super-imposed



Zoom



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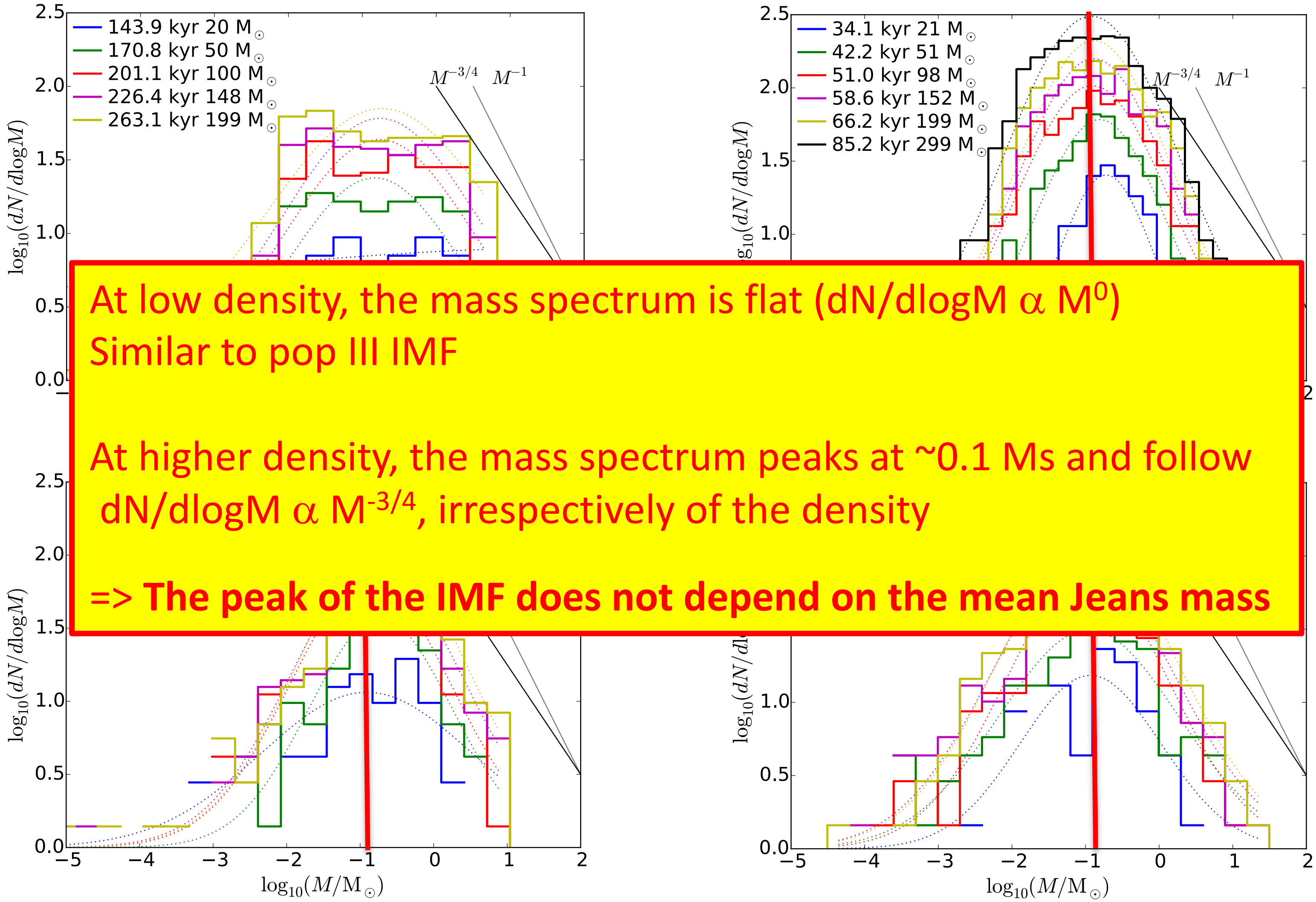
Influence of the initial cloud density on the mass spectrum

Initial density~ 10^3 cm^{-3} (⇔E_turbulent / E_thermal)



At low density, the mass spectrum is flat (dN/dlogM α M^0)
Similar to pop III IMF

At higher density, the mass spectrum peaks at ~0.1 Ms and follow dN/dlogM α $M^{-3/4}$, irrespectively of the density

=> **The peak of the IMF does not depend on the mean Jeans mass**

Initial density ~10^7 cm^{-3}

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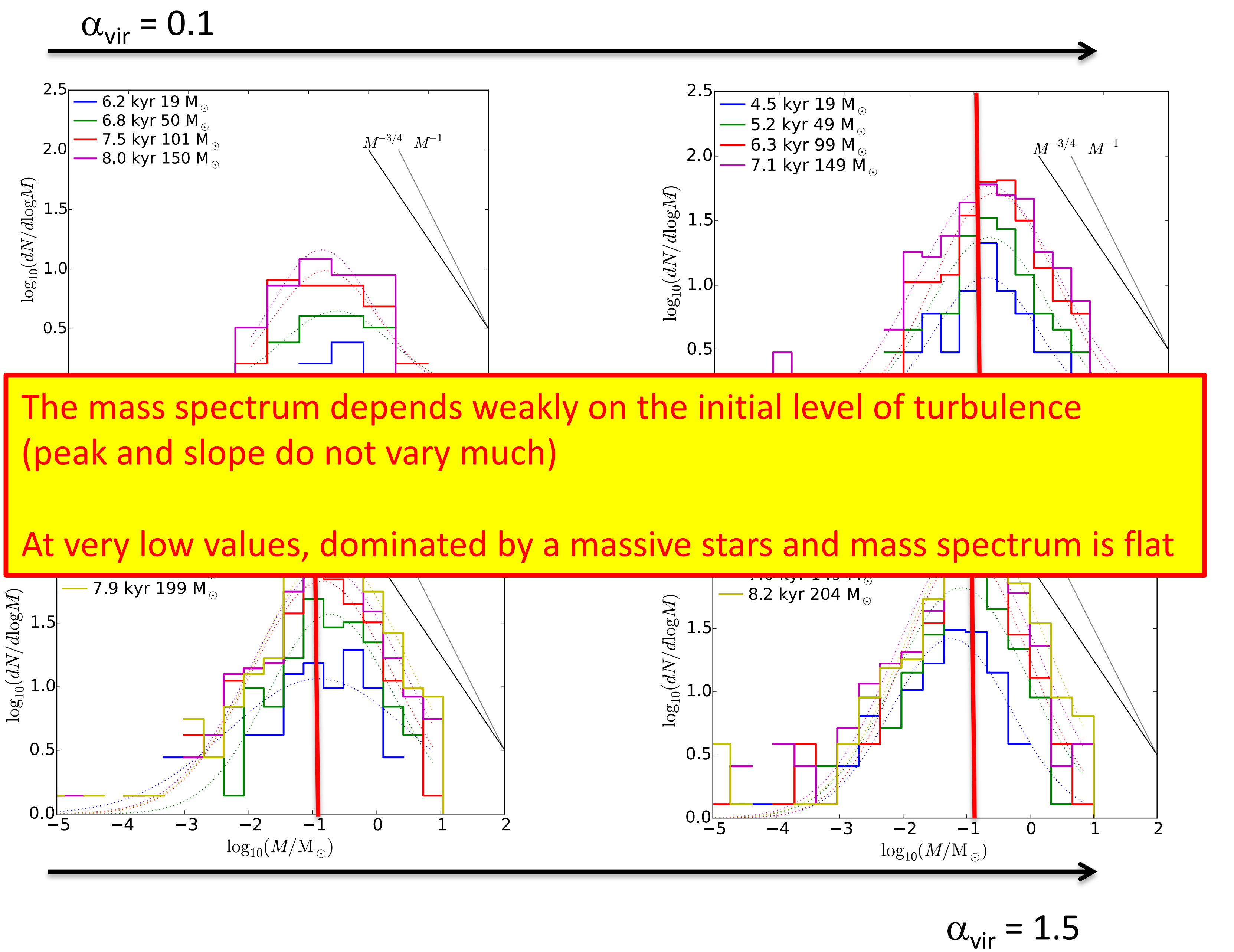
Influence of the initial cloud turbulence on the mass spectrum



The mass spectrum depends weakly on the initial level of turbulence
(peak and slope do not vary much)

At very low values, dominated by a massive stars and mass spectrum is flat

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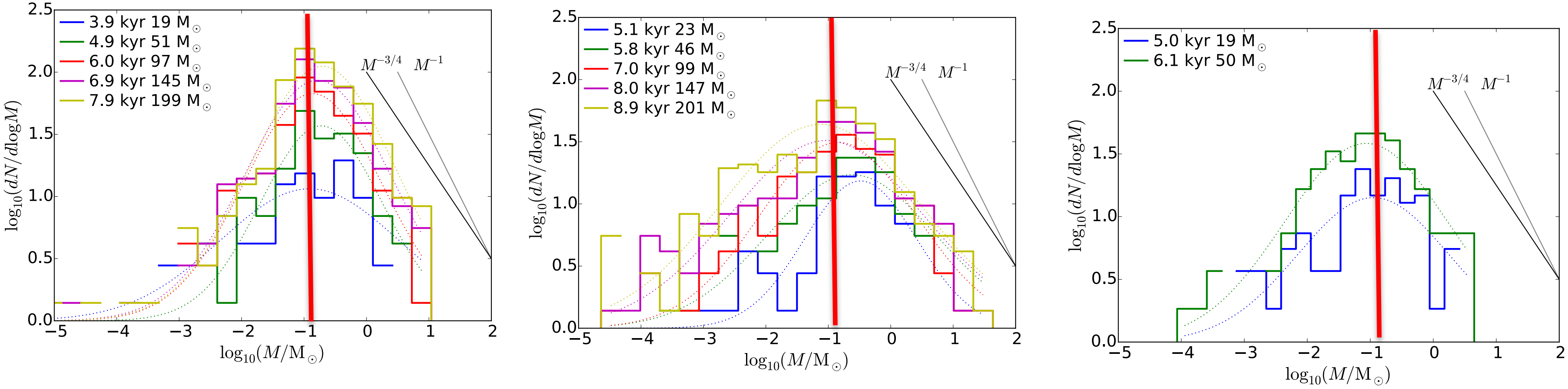
Influence of the initial magnetisation on the IMF

The mass spectrum depends weakly on the initial level of magnetisation (peak and slope do not vary)



β=inf
$\mathcal{M}$=inf

β=0.25
$\mathcal{M}$=5

β=0.125
$\mathcal{M}$=2.5

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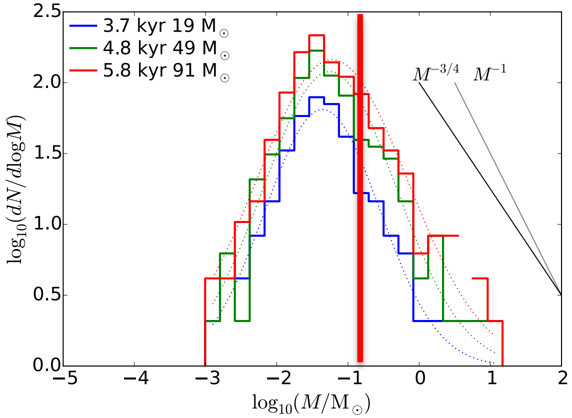
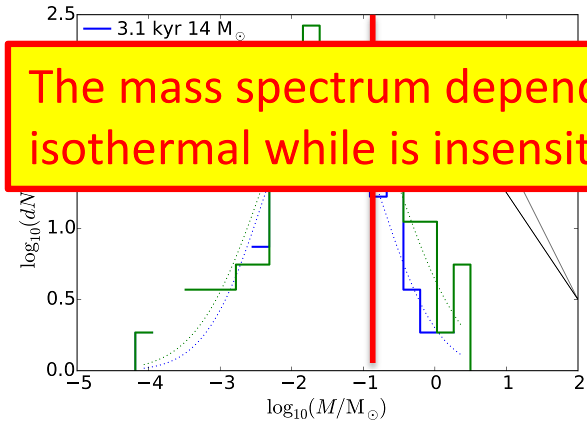
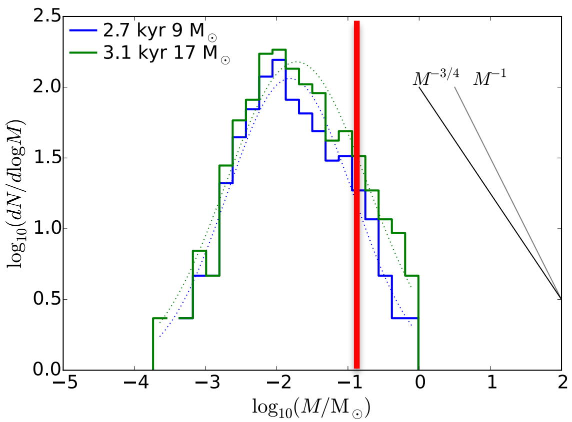
Influence of the numerical resolution on the mass spectrum

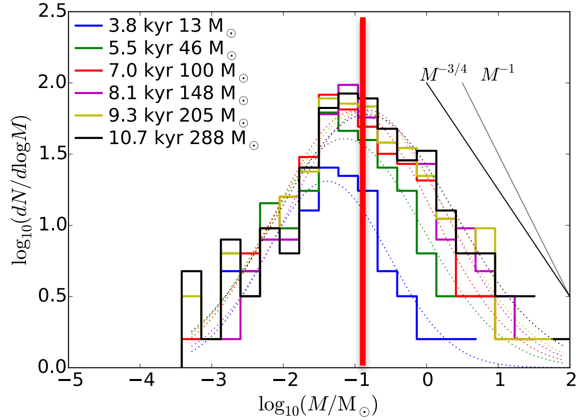
Isothermal



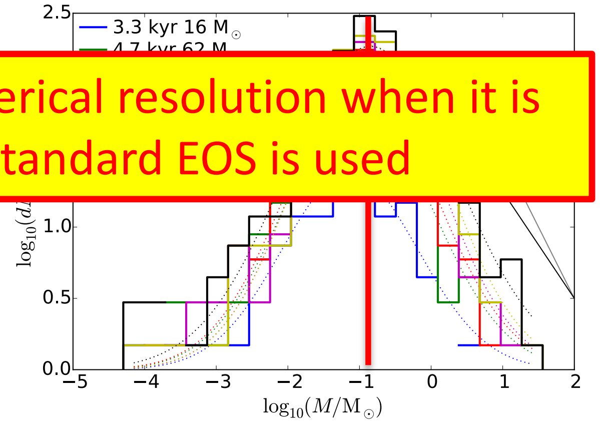




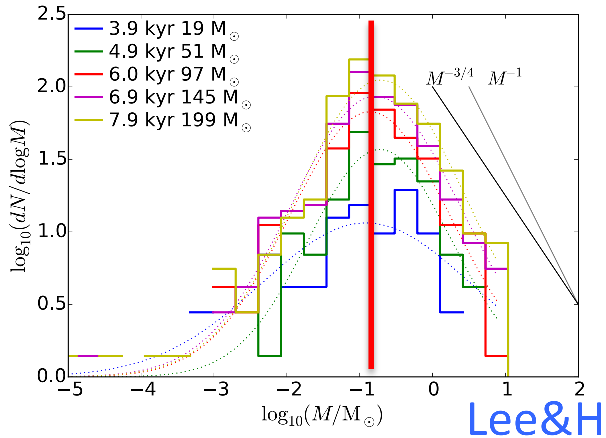
Standard EOS



17 AU



The mass spectrum depends critically on numerical resolution when it is isothermal while is insensitive to it when the standard EOS is used



4 AU

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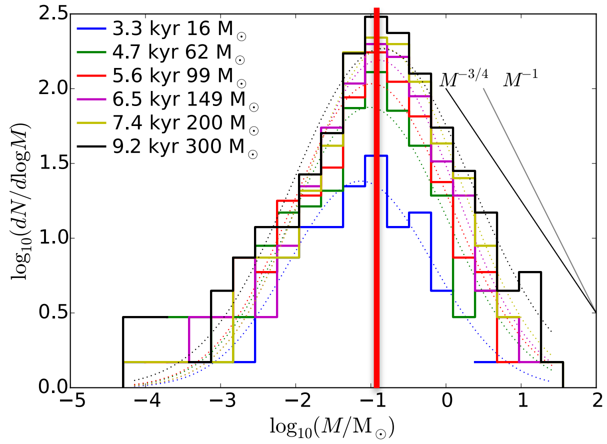
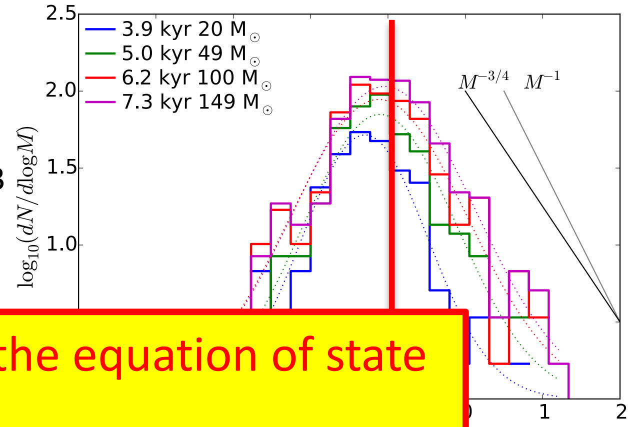
Influence of the EOS on the mass spectrum $T=T_0\,(1 + (n/n_{ad})\,)^{\gamma}$

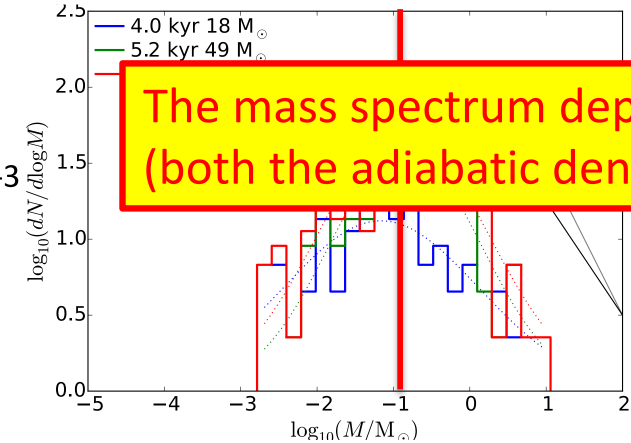
$n_{ad}=10^{10}\,cm^{-3}$
$\gamma=5/3$



$n_{ad}=10^{10}\,cm^{-3}$
$\gamma=4/3$

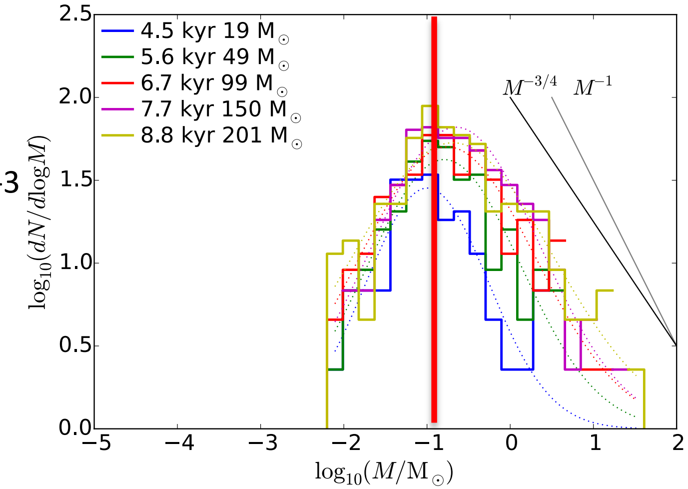


$n_{ad}=3\ 10^{9}\,cm^{-3}$
$\gamma=5/3$

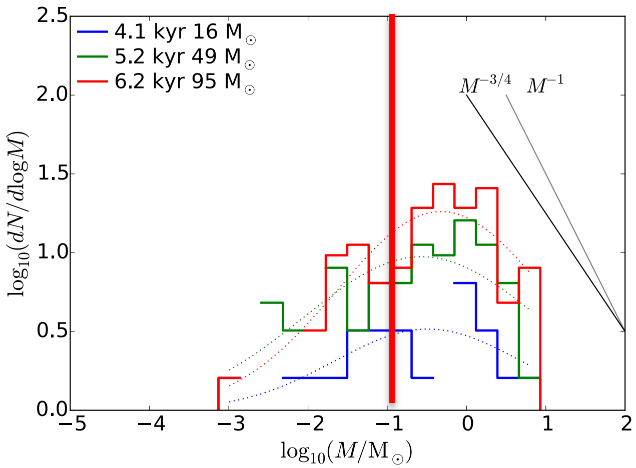


The mass spectrum depends critically on the equation of state (both the adiabatic density and the index)

$n_{ad}=10^{9}\,cm^{-3}$
$\gamma=4/3$



$n_{ad}=10^{9}\,cm^{-3}$
$\gamma=5/3$



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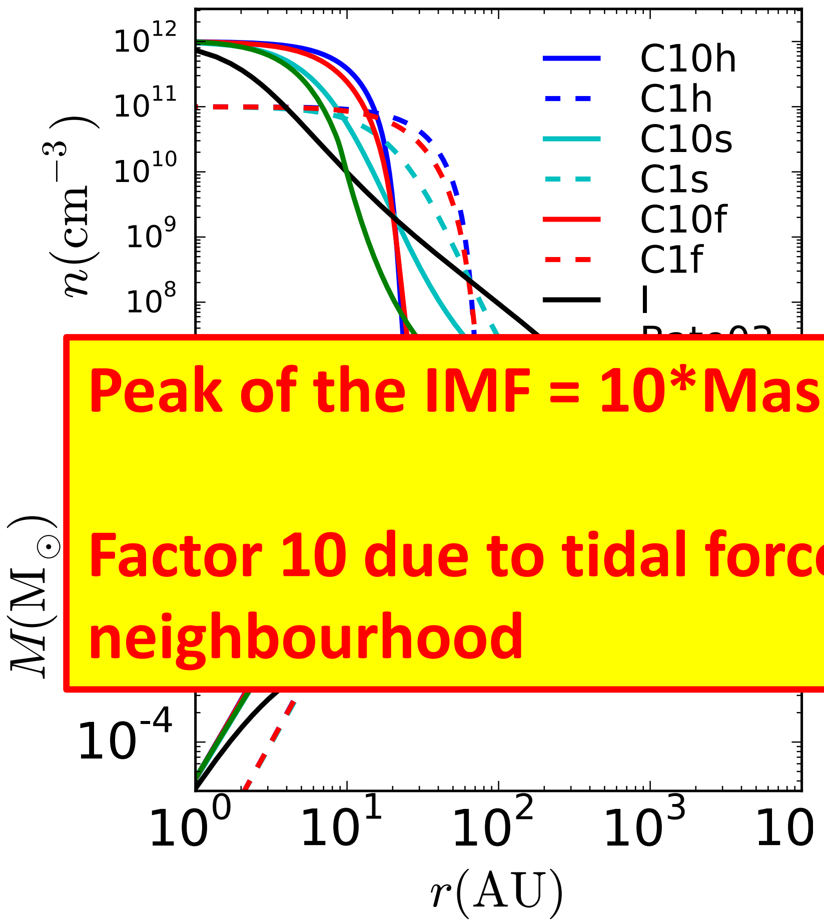
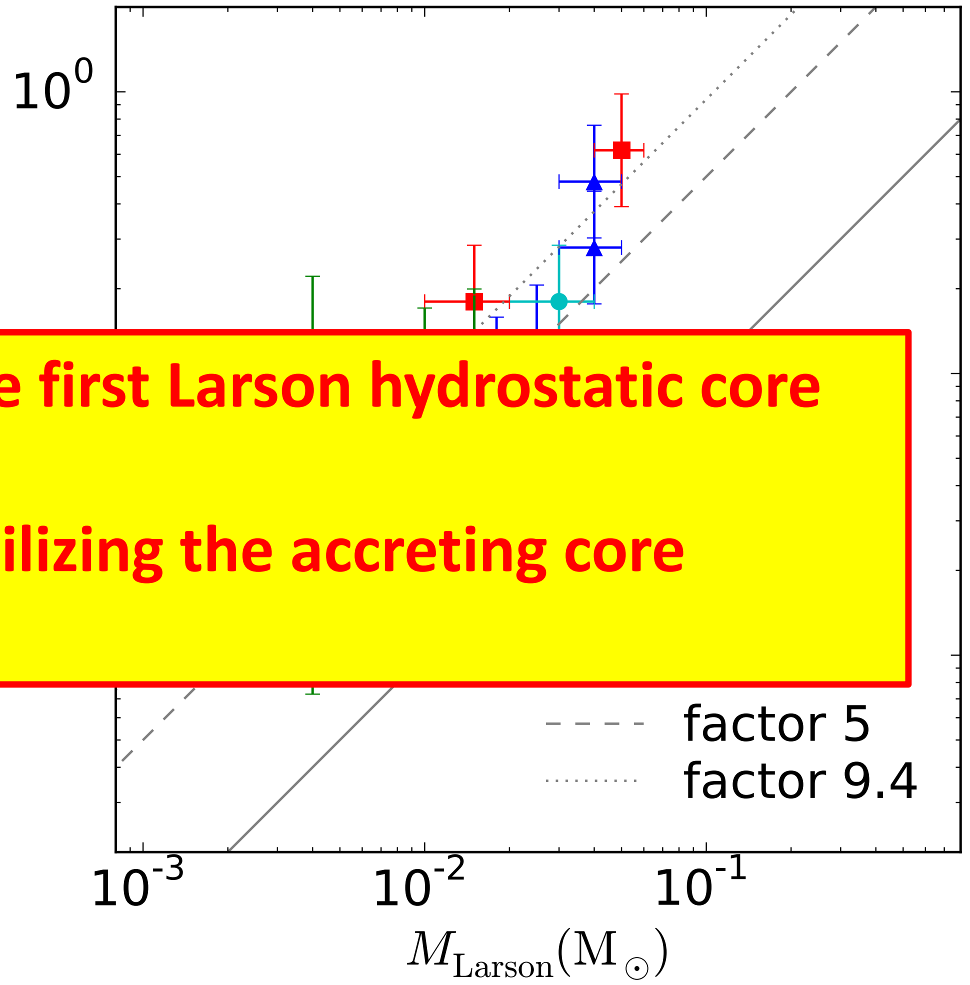
Direct link between the peak of the mass spectrum and the mass of the first Larson core

Density profile of the first Larson core



Mass of the peak vs Mass of the first Larson core



Peak of the IMF = 10*Mass of the first Larson hydrostatic core

Factor 10 due to tidal forces stabilizing the accreting core neighbourhood

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Summary: what can we conclude from these *numerical experiments* ?

Mass spectrum presents two parts:

-a power-law with index of about 0 or ~-1 (> -1)
0 at very low density
>-1 otherwise

-a robust peak close to 0.1 Ms for standard EOS
and more generally 10*Mass of Larson core

These two aspects are important and need to be understood

Analytical approach to understand the mass spectrum

(H& Chabrier 2008)

Balance of unstable mass through scales

$$\frac{M_{\rm tot}(R)}{V_{\rm c}} = \int_{\delta_R^c}^{\infty} \bar{\rho} \exp(\delta) \mathcal{P}_R(\delta) d\delta = \int_0^{M_R^c} M' \mathcal{N}(M') P(R, M')\, dM',$$

=>Mass spectra :
(depends on density PDF and mass-size relation)

$$\mathcal{N}(M_R^c) = \frac{\bar{\rho}}{M_R^c} \frac{dR}{dM_R^c} \left(-\frac{d\delta_R^c}{dR} \exp(\delta_R^c) \mathcal{P}_R(\delta_R^c) \right).$$

Virial theorem:
Condition for unstability

$$M > M_J = a_J \frac{\left[(c_s)^2 + (V_0^2/3)(R/1\mathrm{pc})^{2\eta} \right]^{\frac{3}{2}}}{\sqrt{G^3 \bar{\rho} \exp(\delta)}},$$

$$M = C_m \rho R^3,$$

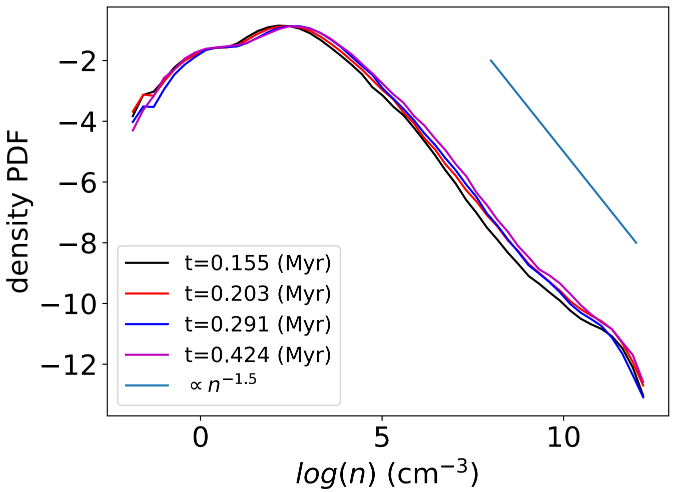
=>Mass-size relation

$$M > M_R^c = a_J^{\frac{2}{3}} C_m^{\frac{1}{3}} \left(\frac{(c_s)^2}{G} R + \frac{V_0^2}{3\,G} \left(\frac{R}{1\mathrm{pc}} \right)^{2\eta} R \right),$$

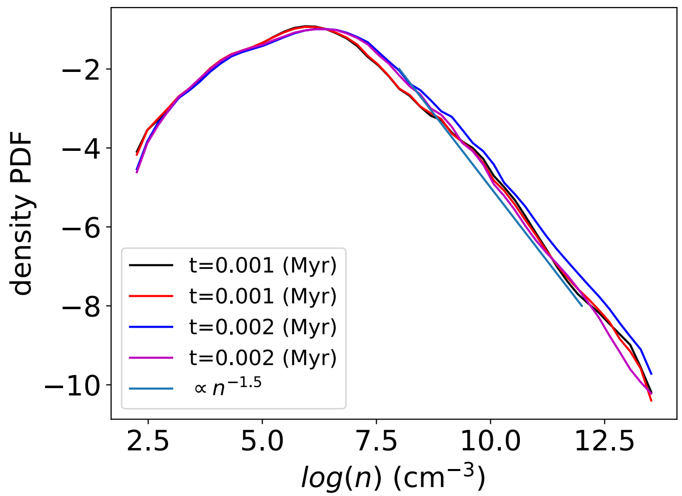
Density PDF within collapsing clouds

(e.g. Kritsuk+2011, H& Falgarone 2012)

Initial density 10^3 cm^{-3}



Initial density 10^7 cm^{-3}



$$\mathcal{P}(\rho) = \mathcal{P}_0 \left(\frac{\rho}{\rho_0}\right)^{-1.5},$$

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Analytical prediction for the mass spectrum with a powerlaw PDF

$$\mathcal{P}(\rho) = \mathcal{P}_0 \left(\frac{\rho}{\rho_0}\right)^{-1.5},$$

$$M > M_R^c = a_J^{\frac{2}{3}} C_m^{\frac{1}{3}} \left(\frac{(c_s)^2}{G} R + \frac{V_0^2}{3\,G} \left(\frac{R}{1\text{pc}}\right)^{2\eta} R \right),$$

Asymptotic behaviour: $M \propto R^{1+2\eta}$,

$$\mathcal{N}(M) \propto \frac{\sqrt{\rho}^{-1}}{M^2} \propto \frac{1}{M^2} \exp\left(-\frac{\delta}{2}\right) \propto M^{-(1+5\eta)/(1+2\eta)}.$$

Thermal support dominates : $\eta = 0$

Turbulent support dominates : $\eta \simeq 0.5$

$$dN/d\log M \propto M^0 \qquad dN/d\log M \propto M^{-3/4}$$

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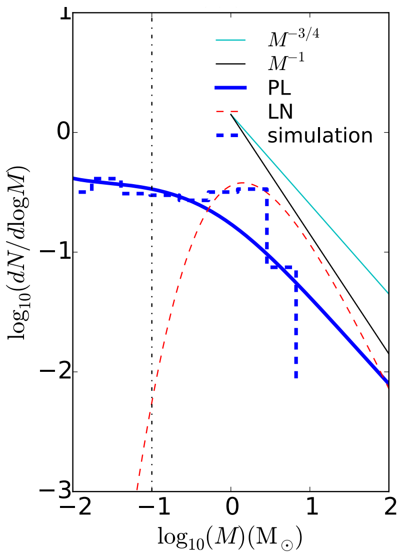
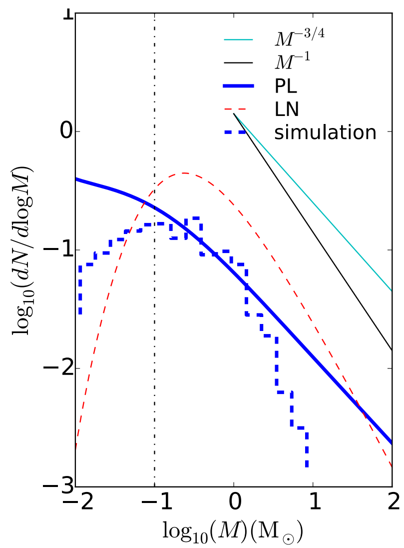
Comparing the analytical prediction and the simulation results: mass spectrum

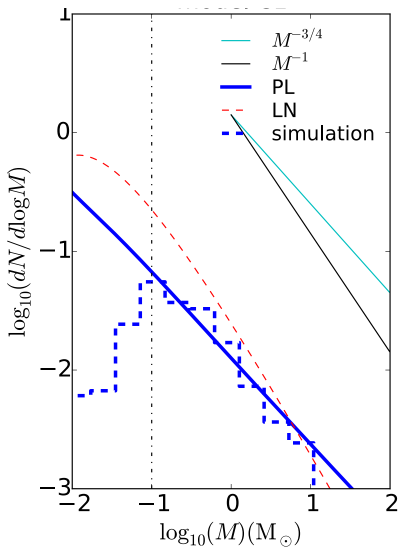
Initial density 10^3 cm^{-3}



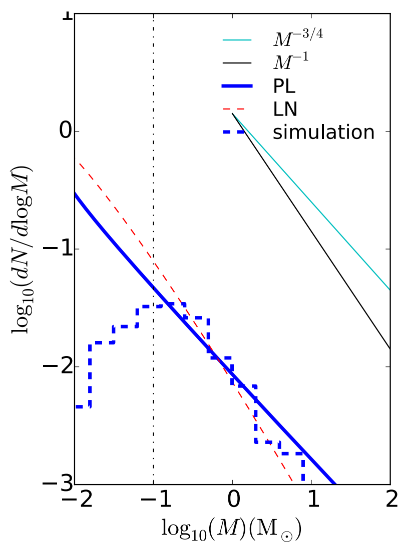
Initial density 3 10^4 cm^{-3}



Initial density 10^6 cm^{-3}



Initial density 10^7 cm^{-3}



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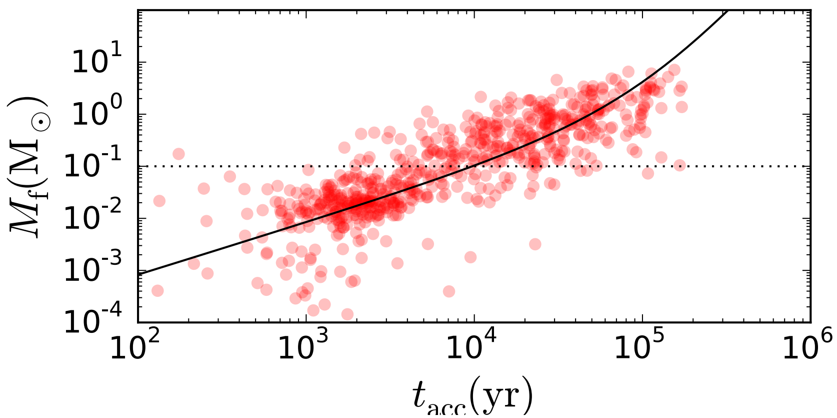
Comparing the analytical prediction and the simulation results: accretion timescale

$$\tau_{\rm ff} = \sqrt{3\pi/(32\rho G)}, \rightarrow \tau_{\rm ff} \propto M^{\frac{1-\eta}{1+2\eta}}$$

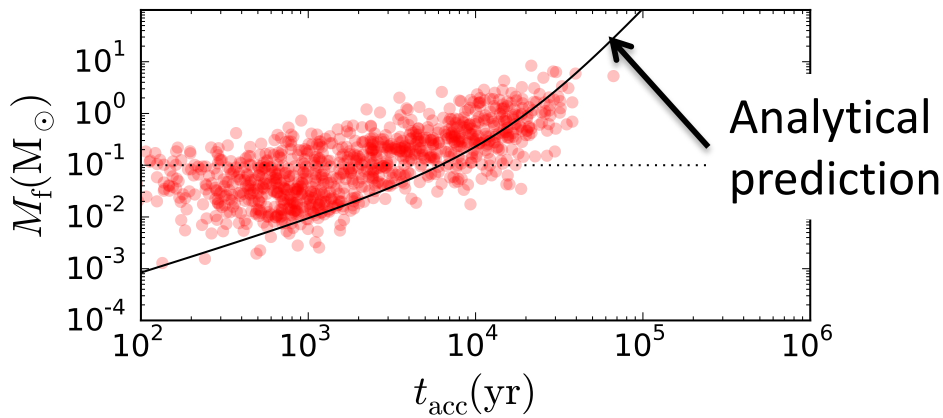
Thermal support dominates : $\eta = 0$ $M \propto \tau_{\rm ff}$

Turbulent support dominates : $\eta \simeq 0.5$ $M \propto \tau_{\rm ff}^4$

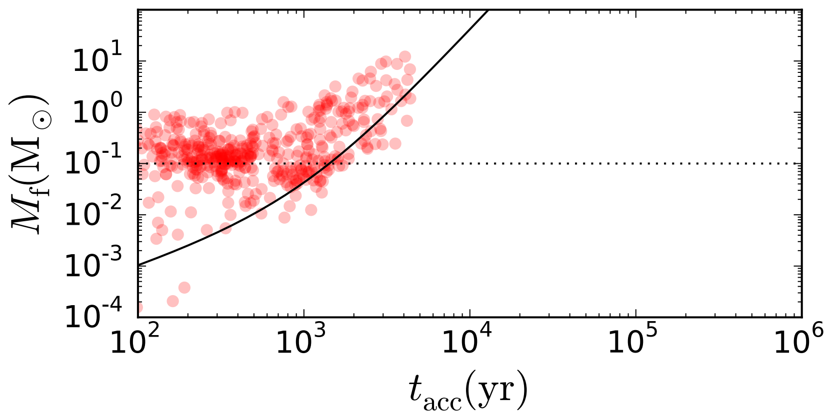
Initial density 10^3 cm^{-3}



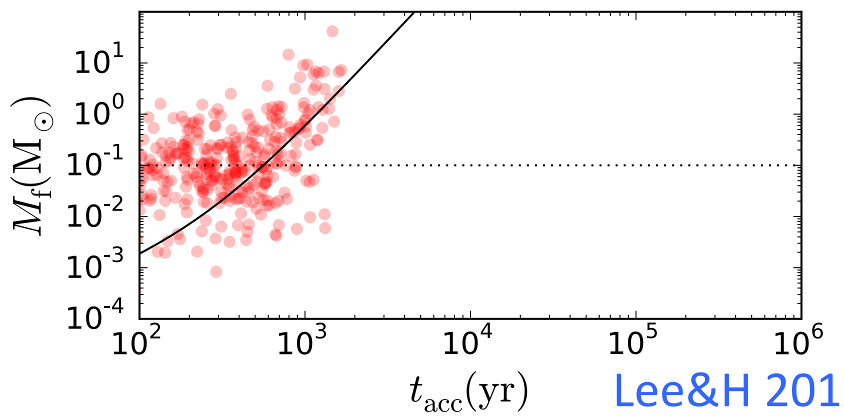
Initial density 3 10^4 cm^{-3}



Initial density 10^6 cm^{-3}



Initial density 10^7 cm^{-3}



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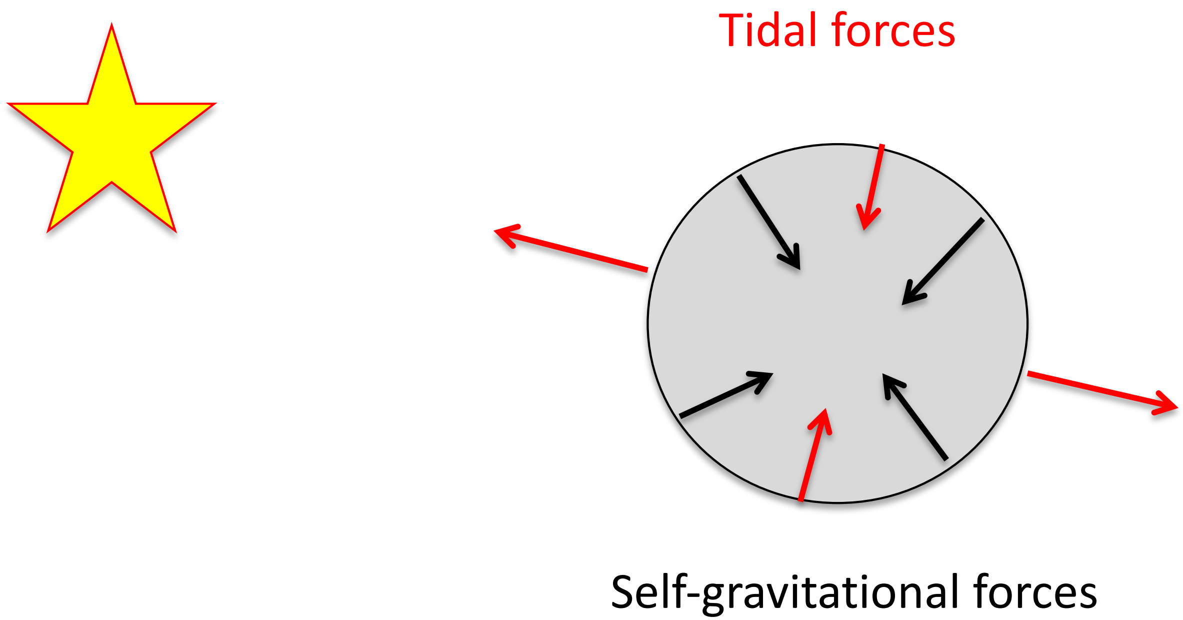
The critical role of tidal forces



For a perturbation to become unstable, self-gravity needs to supersede tidal forces (due to central object and accreting envelope) as well as thermal pressure

The critical role of tidal forces in equations

Density of the envelope

$$\rho_e = \frac{Ac_s^2}{2\pi G}\frac{1}{r_e^2},$$

Density of the perturbation

$$\rho_p = \eta\frac{Ac_s^2}{2\pi G}\frac{1}{r_p^2} \quad \text{for} \quad |\boldsymbol{r}_e - \boldsymbol{r}_p| = |\delta\boldsymbol{r}| = \delta r \le \delta r_p,$$

2 conditions for the perturbation to be unstable

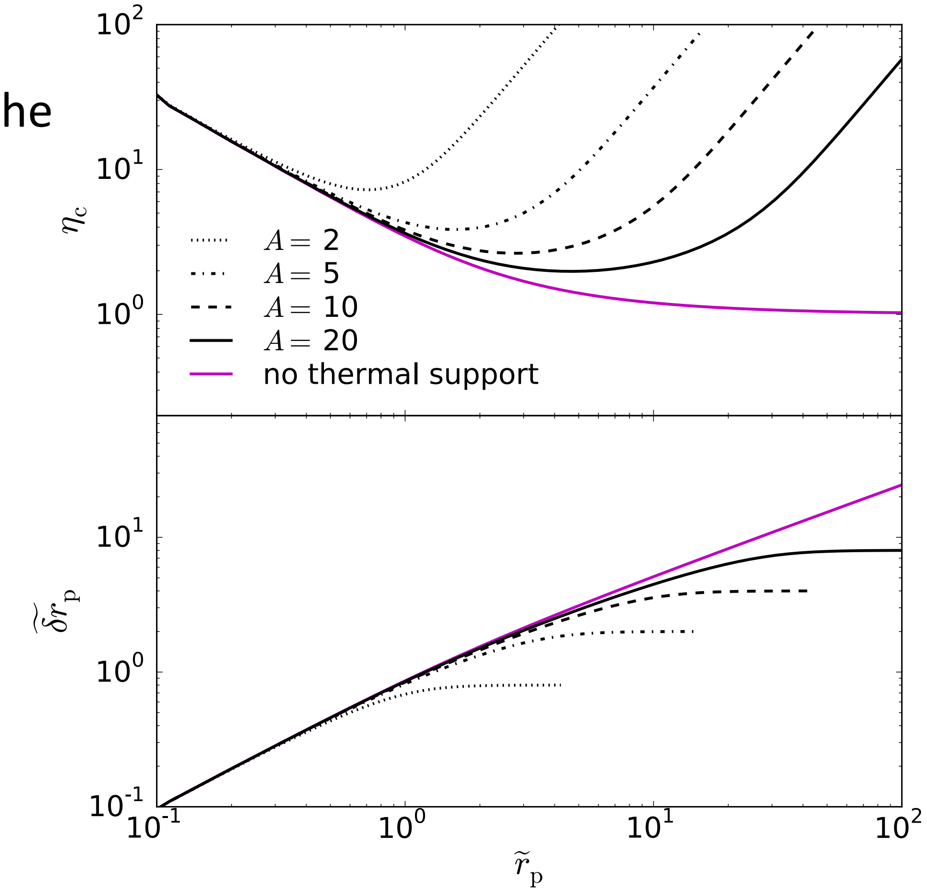
$$M_p(r_p, \delta r_p, \eta) \ge M_L$$
$$E_{vir}(r_p, \delta r_p, \eta) \le 0.$$

$$\begin{aligned} E_{vir}(r_p, \delta r_p, \eta) &= E_g(r_p, \delta r_p, \eta) + 2E_{ther} \\ &= \int_{V_p} \rho \boldsymbol{g} \cdot \delta\boldsymbol{r}\, dV + 3M_p(r_p, \delta r_p, \eta)c_s^2 \\ &= \int_{V_p} (\rho_e + \rho_p)\,(\boldsymbol{g}_L + \boldsymbol{g}_e + \boldsymbol{g}_p) \cdot \delta\boldsymbol{r} dV + 3M_p(r_p, \delta r_p, \eta)c_s^2. \end{aligned}$$

The critical role of tidal forces: results

Minimum amplitude of the unstable perturbations

Radius of the unstable perturbations



Distance from the central object

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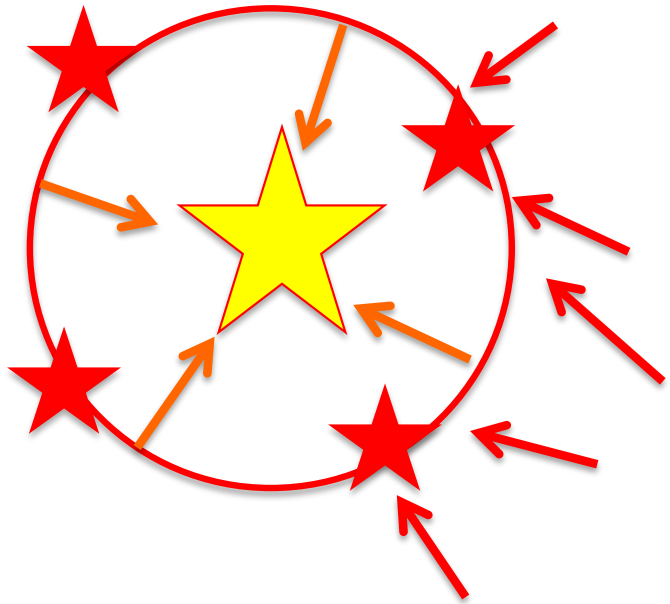
The link between the peak of the IMF and the mass of the first Larson core

The gas inbetween the central object and the first fragment is accreted by the central fragment



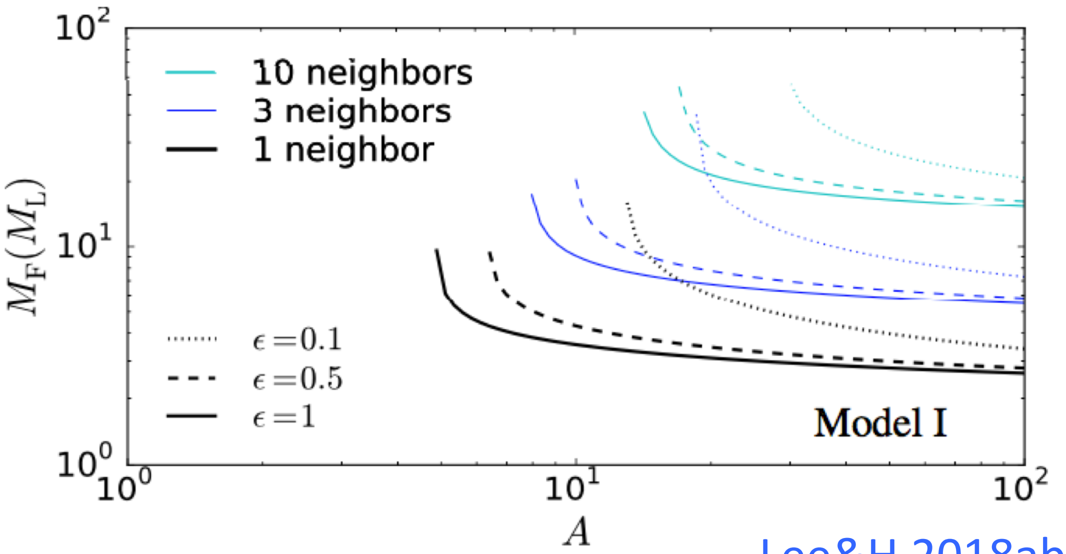
The mean number of fragments at radius r is:

$$\mathcal{N}(\widetilde{r}_p) = \frac{1}{M_L} \int_0^{r_p} \int_{\delta_c}^{\infty} P(\widetilde{r}'_p, \delta)\rho(\widetilde{r}'_p) \exp(\delta) 4\pi r'^2_p \, d\delta \, dr'_p$$

where P is the PDF of the density fluctuations (on top of the $1/r^2$ mean density)

The final mass of the central fragment is the mass inside the sphere of radius r_p

$$M_e(r_p) = 2Ac_s^2 r_p / G$$



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Conclusions

During the collapse of a massive clouds a IMF-like distribution of stars forms

-a power-law with index -3/4 (or 0 if thermal support is too high)

-a peak that depends only on the EOS and **M_{peak} ~ 10* M_{larson}**

General proposition regarding the IMF:

The peak of the IMF is universal because it is *local* physics (EOS, collapse, tidal forces)

There are “probably” 2 powerlaw regimes:
-Salpeter-like (-1.3) due to turbulent dispersion and gravity in a lognormal PDF density
-more shallow (-3/4) due to turbulent dispersion and gravity in a powerlaw PDF

Analytical prediction for the mass spectrum with a lognormal PDF

$$\mathcal{P}(\delta) = \frac{1}{\sqrt{2\pi\sigma_0^2}} \exp\left(-\frac{(\delta-\bar{\delta})^2}{2\sigma_0^2}\right),$$

where $\bar{\delta} = -\sigma_0^2/2$ and $\sigma_0^2 = \ln(1 + b^2\mathcal{M}^2)$.

Asymptotic behaviour: $M \propto R^{1+2\eta}$,

$$N(M) \propto \frac{\sqrt{\rho}}{M^2} \propto M^{-3(1+\eta)/(1+2\eta)}$$

Thermal support dominates : $\eta = 0$

$$dN/d\log M \propto M^{-2}$$

Turbulent support dominates : $\eta \simeq 0.5$

$$dN/d\log M \propto M^{-1.25}$$

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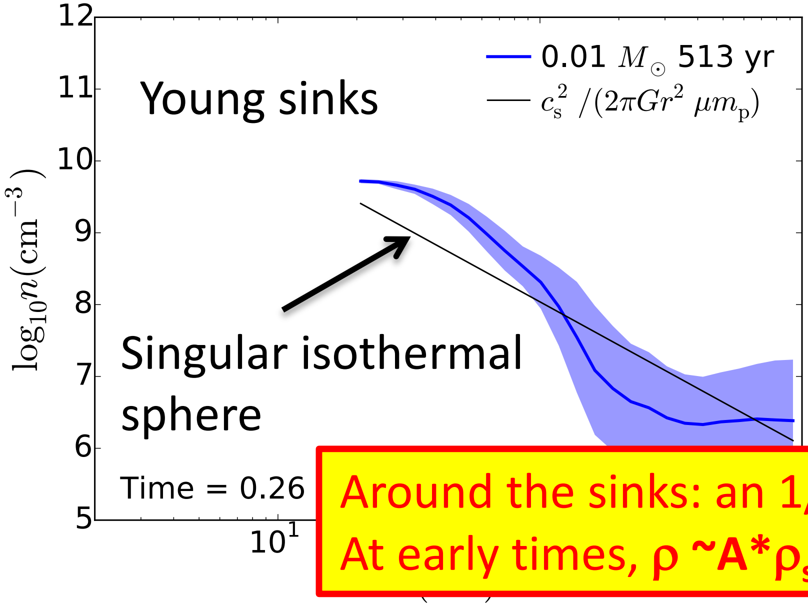
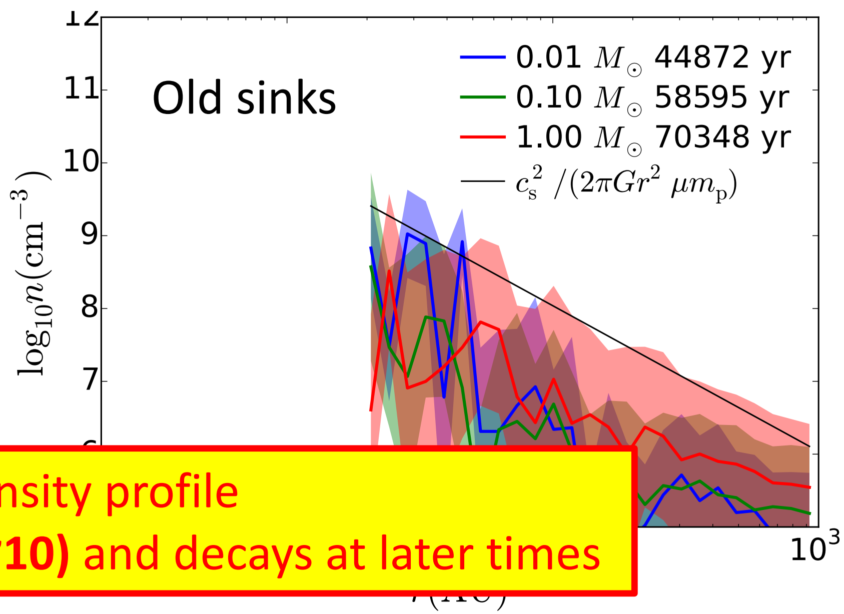
Looking at the surrounding gas around sinks

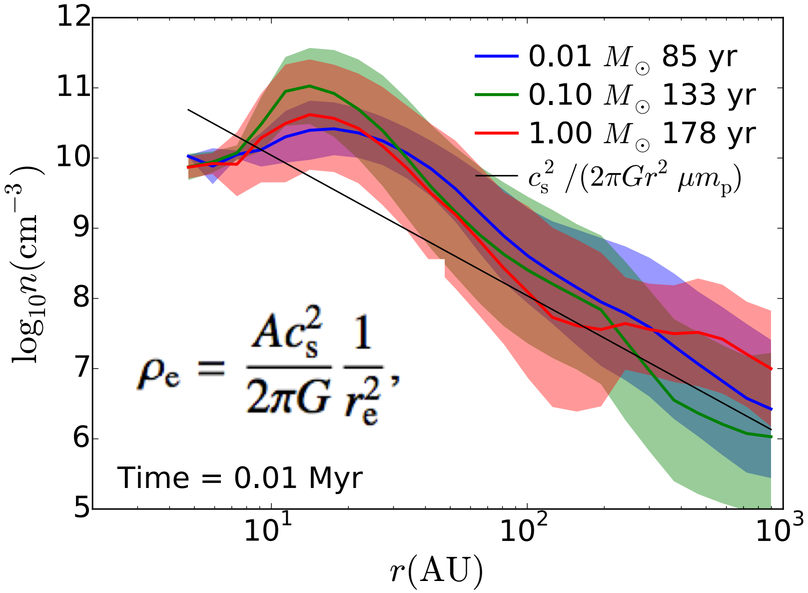
Initial density 10^3 cm^{-3}

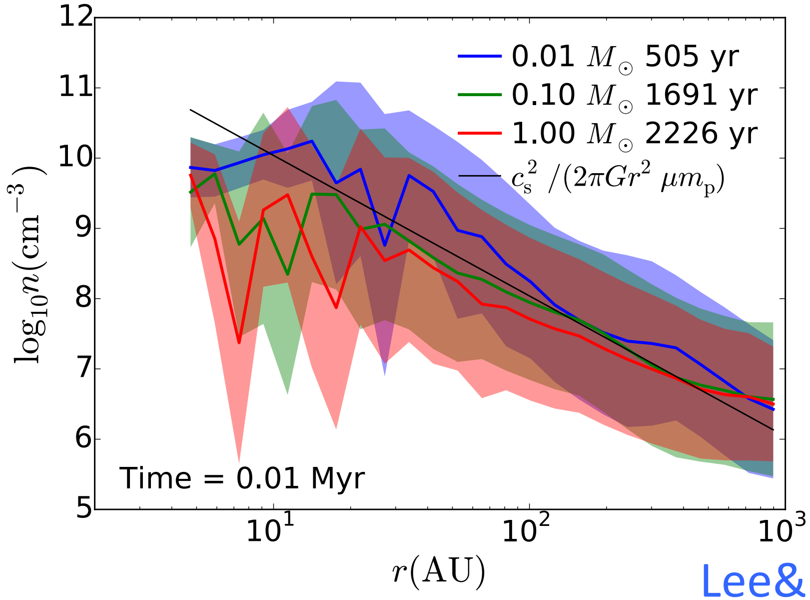




Around the sinks: an $1/r^2$ density profile
At early times, ρ ~$A*\rho_{sis}$ (A~10) and decays at later times

Initial density 10^6 cm^{-3}





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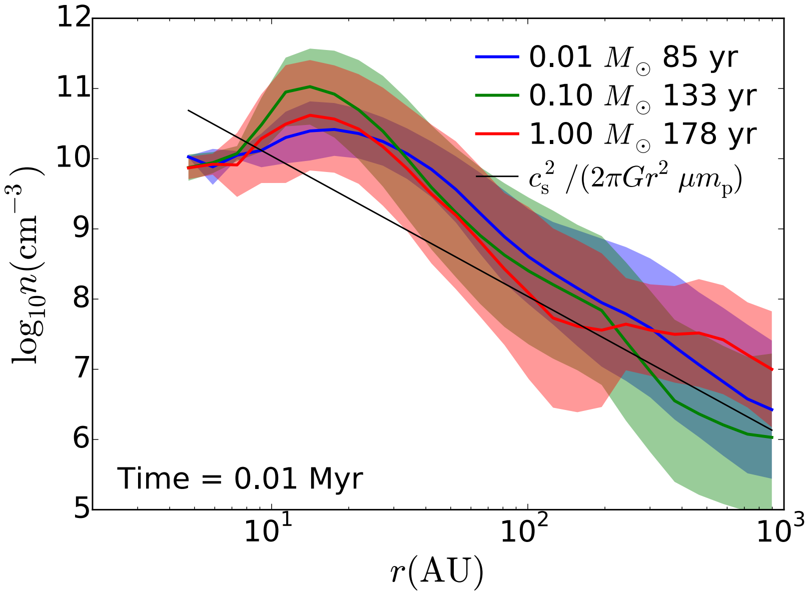
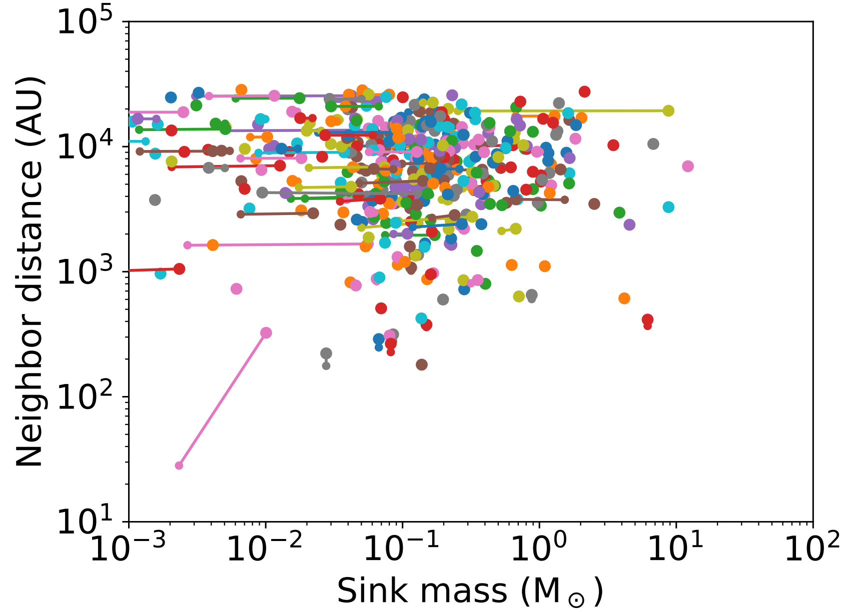
Looking at the surrounding neighbours of the sinks





Very dense gas around the sinks but… neighbours are far !

Something stabilizes the gas in the vicinity of existing stars/sinks. What is it ?

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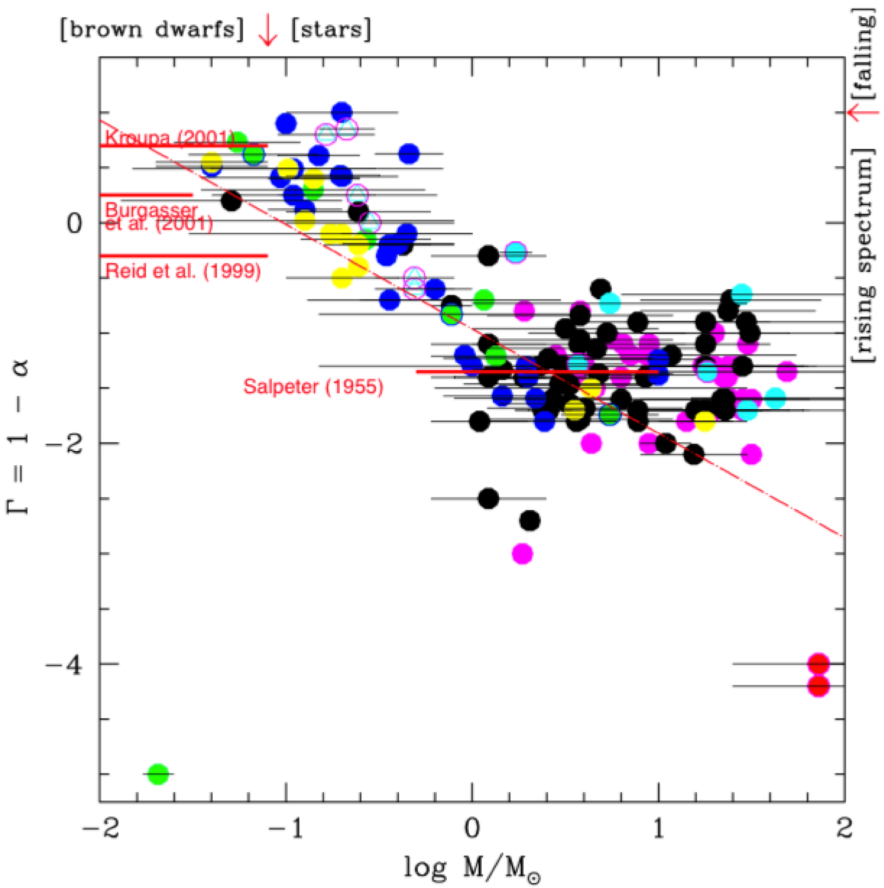
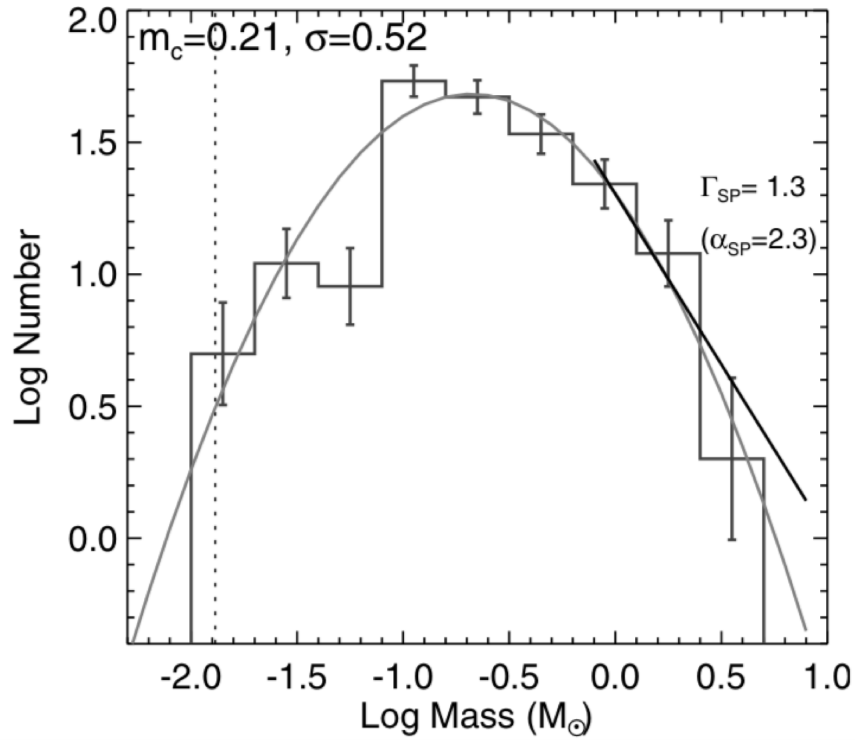
The Initial Mass Function

(Salpeter 1955, Kroupa 2002, Chabrier 2003, Hillenbrand 2004, Moraux+2007, Bastien+2010, Offner+2014)



Hillenbrand 2004



Alves de Oliveira 2013